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ON THE ARC-SPECTRA OF THE ELEMENTS OF THE PLATINUM GROUP. I.

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SOME years ago very little was known of the spectra of the elements of the platinum group. For Pt and Pd, Kirchhoff,[1] Huggins,[2] Thalén,[3] and Lecoq de Boisbaudran[4] had measured the strongest lines in the visible part of the spark-spectrum; for Os Huggins[2] gave eighteen lines; for Ir Kirchhoff[1] had found three lines in the spark-spectrum, and Lockyer[5] some lines of the arc-spectrum. Of Rh and Ru we knew nothing.

The first modern research with a Rowland grating was made by McClean.[6] He published photographs of parts of the spark-spectra of the six elements, together with the solar spectrum. Then Rowland[7] gave the first exact determination of the arc-spectra of these elements, excepting Ir, between the wave-lengths 3000 and 4500. Lastly Exner and Haschek[8] have published the ultra-violet spark-spectra of the six elements, photographed with a small Rowland grating, but measured with less accuracy than is desirable.

The determination of these spectra is exceptionally difficult because of the impossibility of preparing the elements suffi-

[1] KIRCHHOFF, *Untersuchungen ueber das Sonnenspectrum*, Berlin, 1861.
[2] HUGGINS, *Phil. Trans.*, **154** (1864).
[3] THALÉN, *Nova Acta. R. Soc. Sc. Upsala* (3), **6** (1868).
[4] LECOQ DE BOISBAUDRAN, *Spectres Lumineux*, Paris, 1874.
[5] LOCKYER, *Phil. Trans.*, 1881.
[6] MCCLEAN, *Comparative Photographic Spectra of the Sun and the Metals*, 1893 (see *M. N.*, No. 52).
[7] ROWLAND, *Ap. J.*, **2** (1895) and **3** (1896).
[8] EXNER and HASCHEK, *Sitz. d. Wien. Akad.*, **104**, 2 (1895), **105** (1896).

ciently pure and free from traces of one another. I had the fortune to get, through the kindness of Dr. Bettendorff, in Bonn, different salts of these elements, which he had prepared some years ago with all possible care, with the purpose of a new determination of the atomic weights. They were incomparably purer than the material to be had in commerce, and I hope that by their use I have been able to ascribe most lines to the elements to which they really belong.

The wave-lengths were determined with the aid of iron lines. For that purpose I have first measured the iron spectrum. On the basis of Rowland's[1] standards of the iron spectrum in the arc, not in the Sun, all of the stronger lines between 2300 and 4500 were determined with a mean error of, say, 0.003 A. U. For greater wave-lengths than 4500 it is impossible to interpolate with sufficient accuracy between the very few lines that Rowland has given. For this part of the spectrum I have therefore taken as standards the iron lines measured by Rowland[2] in the solar spectrum. I am well aware that this is inaccurate, but the shift between the solar lines and the arc lines will be a small one, and when it has once been determined, it will be easy to apply a correction to my numbers.

The wave-length of every line of the platinum elements is the mean of at least four determinations made on at least two different plates. The mean error of all the final values, excepting the diffuse lines, lies between 0.000 and 0.005 A. U. I think an error of 0.010 A. U. will very seldom occur.

The agreement between Rowland's wave-lengths and my own is very satisfactory. For Os, *e. g.*, we have 141 lines in common. For 73 of these lines the difference is between 0.000 and 0.005 A. U.; for 37 lines between 0.005 and 0.010; for 27 lines the difference is greater than 0.010 A. U. Rowland has measured only the stronger lines of the elements, so that my tables contain many more lines. A small number of Rowland's lines I have not found. This is partly due to the overlapping of the cyanogen bands, which seem to have been stronger on my plates than on Rowland's, so that I have not seen some lines lying in

[1] ROWLAND, *A. and A.*, **12**, 1893. [2] ROWLAND, *Ap. J.*, **1** and **2** (1895).

them. The other lines not found by me I ascribe to impurities, for in the spectrum of Ru Rowland gives seven lines belonging to Zn.

In the following tables the intensities are given in a scale from 0 (weakest line) to 10 (strongest line). The self-reversed lines, denoted by r, are estimated from 3 to 10. u means diffuse (unscharf), U very diffuse (sehr unscharf), (such lines occur in great number in the spectrum of Pd), and uR or uV means diffuse on the side of longer and shorter wave-lengths, respectively.

I. PLATINUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2305.72	2	2450.527	2	2544.042	4
2308.12	3	2451.046	3	2544.807	2
2315.58	2	2460.160	1	2546.562	0
2318.371	2	2461.474	0	2546.986	0
2326.185	2	2467.504	6, r	2548.194	0
2331.047	1	2469.537	0	2549.552	3
2340.255	2	2471.092	3	2552.326	3
2343.468	0	2473.247	0	2560.438	0
2346.822	0	2477.365	0	2564.263	0
2347.239	0	2481.270	2	2572.723	0
2353.123	0	2483.312	2	2574.580	2
2356.415	0	2483.452	2	2582.415	2
2357.181	4, r	2487.261	4, r	2587.890	2
2357.656	0	2488.819	4	2596.081	4
2368.357	4, r	2490.217	2	2599.148	0
2380.035	0	2495.910	4	2599.986	2
2383.732	4	2497.197	1	2602.182	0
2386.886	0	2498.592	4	2603.223	4
2387.448	0	2500.895	0	2606.126	0
2389.615	3	2503.075	2	2608.333	0
2391.856	0	2504.128	2	2613.204	0
2396.243	2	2506.014	4	2613.337	0
2396.762	0	2508.589	3	2614.701	2
2401.089	1	2510.604	0	2616.839	0
2401.959	3	2513.999	0	2619.668	4
2403.180	4, r	2514.165	2	2619.977	0
2413.138	1	2515.119	3	2625.419	2
2418.151	3	2515.666	3	2627.484	4
2420.912	0	2517.273	1	2628.122	7, r
2424.964	2	2520.356	0	2635.372	0
2426.523	2	2522.616	0	2639.434	5
2428.206	8, r	2529.499	2	2645.453	4
2429.186	2	2536.068	2	2646.969	6, r
2434.551	0	2536.581	4	2650.938	4, r
2436.771	4, r	2538.361	0	2653.867	0
2439.533	1	2539.285	3	2656.907	0
2440.158	4, r	2541.433	2	2658.266	4

PLATINUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2658.790	2	2788.728	0	2901.798	0
2659.535	10, r	2790.593	0	2903.129	0
2664.723	2	2790.987	0	2904.258	0
2668.748	0	2793.372	4	2906.001	4
2673.707	0	2793.736	2	2908.008	4
2674.649	4	2794.304	5	2808.928	0
2677.232	5, r	2796.165	1	2910.569	3
2686.990	0, u	2800.560	0	2911.888	0
2688.352	2	2803.338	6	2912.884	0
2694.314	4	2806.151	0	2913.361	2
2696.069	0	2807.396	0	2913.655	4
2698.498	6	2808.603	4	2914.443	0
2701.208	0	2810.921	0	2915.278	0
2702.484	6, r	2813.080	2	2916.505	2
2705.985	5, r	2814.121	0	2919.451	4
2713.215	4	2818.354	4	2921.336	1
2714.613	0	2818.741	2	2921.498	3
2715.866	2	2821.179	0	2922.381	0
2717.709	0	2822.273	0	2927.040	1
2719.125	6, r	2822.602	2	2928.226	4
2725.433	2	2825.192	1	2929.903	8, r
2730.002	5	2830.402	8, r	2830.904	4
2733.725	5, r	2831.981	0	2933.837	0
2734.057	8, r	2834.815	0	2938.935	4
2734.584	2	2837.338	2	2941.219	2
2736.886	0	2837.643	0	2941.908	0
2737.656	2	2839.345	2	2942.880	4
2738.569	4	2848.406	0	2944.879	3
2744.928	2	2849.241	1	2948.844	0
2747.701	4	2853.207	4	2949.900	2
2753.850	2	2853.484	2	2950.929	0
2753.957	3	2854.781	0	2951.341	2
2754.327	0	2855.866	0	2958.650	0
2755.003	4	2868.783	0	2959.219	4
2757.799	2	2870.572	4	2959.825	1
2758.164	0	2878.823	1, u	2960.864	5, u
2758.333	2	2884.583	1	2967.596	0
2759.424	0	2885.447	0	2969.965	0
2763.299	0	2888.307	4	2974.252	0
2766.764	5	2890.495	2	2978.179	2
2769.940	4	2891.030	2	2982.414	0
2771.750	4, r	2891.170	0	2983.882	2
2772.925	4	2891.873	0	2984.565	0
2773.696	2	2893.335	4	2988.177	0
2774.095	4	2893.984	6	2988.913	0
2774.306	3	2896.245	1	2989.915	4
2774.880	2	2897.988	5	2994.916	2
2776.111	1	2899.764	1	2998.087	7, r
2776.859	0	2900.903	0	3001.304	2
2777.558	0	2901.282	2	3002.385	4

PLATINUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3003.400	2	3104.170	0	3252.785	1
3004.269	2	3112.718	0	3253.319	0
3005.911	2	3118.547	0	3255.356	0
3010.051	0	3119.911	4	3256.048	6
3012.498	2	3122.192	0	3256.634	1
3014.636	0	3123.065	0	3258.551	0
3015.013	0	3132.187	0	3259.282	1
3015.510	2	3133.443	4	3259.866	4
3017.450	2	3133.785	1	3261.202	2
3018.003	4	3134.413	1	3261.818	4
3019.961	0	3136.381	0	3263.737	1
3022.957	3	3139.503	7	3268.557	4
3024.410	2	3141.767	4	3282.104	5
3025.179	2	3154.858	1	3283.336	2
3025.671	2	3156.686	5	3283.443	2
3026.446	2	3159.841	0	3285.367	0
3036.554	6	3160.314	1	3287.245	0
3039.612	0	3169.006	1	3290.363	6
3041.323	2	3174.959	2	3293.615	0
3042.752	4, r	3176.081	1, u	3293.820	0
3048.6	2, U	3177.707	1	3298.688	0, u
3054.4	2, U	3179.650	1	3300.070	1, u
3054.8	2, U	3191.604	0	3302.015	8, u
3055.402	4	3192.635	3	3311.504	1
3056.719	0	3199.076	0	3311.959	2
3059.748	4	3199.215	0	3312.614	3
3061.905	1	3200.848	4	3313.186	1
3062.3	0, U	3204.165	6	3315.186	4
3062.845	0	3207.347	0	3323.914	6
3064.825	6, r	3208.968	0	3325.861	2
3069.207	2	3212.502	2	3327.234	0
3070.369	2	3218.603	1	3338.214	2
3072.042	5	3218.972	0	3342.429	1
3074.938	1	3220.904	3	3344.031	4
3075.122	0	3221.416	0	3367.139	4
3078.948	0	3222.680	0	3368.628	2
3079.674	4	3222.930	0	3372.960	0
3081.172	0	3223.928	0	3406.733	2
3082.779	0	3227.305	2	3408.286	7
3084.217	3	3230.401	5	3414.610	2
3084.978	2	3233.550	5	3417.227	2
3087.319	0	3240.324	5	3418.311	0
3088.677	0	3241.652	1	3420.493	0
3089.780	0	3243.224	0	3426.887	2
3098.887	0, u	3243.533	2	3428.079	4
3100.146	4	3247.388	2, d ?	3431.495	0
3101.077	4	3248.623	2	3432.002	2
3102.710	0	3248.843	0	3448.523	1, u
3103.231	1	3250.481	4	3454.290	3
3103.704	2	3252.117	5	3464.097	2

PLATINUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3472.080	0	4092.426	3	4831.371	0
3483.588	5	4118.854	5	4854.067	4
3485.430	6	4164.709	4	4862.577	0
3488.877	1	4192.577	4	4879.700	4
3491.155	1	4201.374	2	4980.532	1
3498.321	1	4247.838	1	4998.123	2
3505.848	1	4251.277	1 [1]	5002.762	2
3514.869	4	4263.664	2	5033.686	4
3528.700	2	4269.411	2	5037.859	0, u
3611.057	2	4274.042	2	5038.681	0
3615.443	0	4281.905	1	5044.194	4
3621.839	2	4288.215	4	5044.645	6
3628.275	5	4291.070	2	5050.006	1
3629.025	3	4327.243	4	5059.658	5
3638.956	6	4334.827	2	5095.950	0, u
3643.331	6	4343.852	0	5118.583	1
3652.411	1	4358.522	2, u	5194.050	1
3654.132	1	4364.624	4	5208.775	0
3659.571	2	4391.999	4	5227.782	6
3663.239	4	4411.580	3	5257.609	0, u
3668.564	1	4414.420	2	5260.982	3
3672.165	4	4437.470	4, u	5265.290	0
3674.207	4	4442.730	6	5275,008	0, u
3675.107	1	4445.710	4	5286.289	0, u
3681.227	0	4473.633	3, u	5295.918	0
3683.169	4	4481.808	3	5301.182	6
3687.582	4	4484.882	5, u	5306.493	0
3700.070	4	4493.350	3	5319.540	0
3706.685	3	4498.926	6	5324.799	0
3818.827	5	4511.417	3, u	5369.188	4
3898.880	4	4521.099	5, u	5388.105	2
3900.873	4	4523.192	5, u	5391.010	4
3903.864	2	4548.056	3, u	5452.984	0
3904.534	3	4552.116	2	5469.714	2
3906.433	2	4552.586	5, u	5475.996	6
3911.045	3	4554.759	4	5478.722	6
3923.105	5	4560.209	4	5514.324	4
3925.483	4	4577.584	4	5526.077	4
3948.550	4	4580.685	2	5560.245	2
3953.780	1	4580.828	2	5684.908	2
3966.507	3	4639.982	4	5699.190	1
3976.460	1	4650.192	1	5700.672	0
3980.746	1	4658.105	5	5728.369	0
3996.720	3	4684.255	4	5762.877	3
4002.649	2	4737.722	2	5763.778	3
4054.928	2	4739.924	1	5840.354	5
4066.087	2	4746.046	1	5845.050	4
4081.631	1	4772.467	1	5861.074	2

[1] Rowland.

II. PALLADIUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2316.569	0	2605.157	4	3025.094	0
2319.328	0	2609.716	0	3028.031	4, r
2327.575	0	2614.270	1	3028.894	1
2336.522	0	2631.692	0	3029.600	0
2336.663	0	2641.149	0	3032.324	1
2347.611	0	2653.7	2, U	3036.220	1
2351.428	0	2655.1	4, uR	3038.830	0
2357.732	0	2658.819	2	3039.128	0
2360.614	0	2663.249	2	3039.748	0
2362.406	0	2676.050	2	3041.102	0
2368.044	0	2686.373	3	3046.614	2
2373.701	0	2734.095	0	3049.502	0
2414.850	0	2742.532	2	3062.3	0, u
2418.835	0	2751.972	2	3062.430	0
2421.	3, U	2763.199	8, r	3063.537	1
2424.564	0	2802.009	3	3065.425	4, r
2426.964	0	2806.561	1	3066.210	1
2431.051	0	2807.8	2, uR	3067.243	0
2435.408	0	2835.133	0	3068.564	0
2441.52	6, r, U	2835.385	0	3073.924	0
2446.275	0	2837.2	2, U	3075.274	4
2447.998	10, r	2839.5	4, uR	3078.356	0
2457.361	0	2846.4	2, U	3088.636	0
2461.2	1, U	2849.912	2	3089.756	0
2469.353	0	2854.694	2	3103.176	0
2470.091	0	2875.875	2	3103.909	0
2471.275	0	2922.615	7, r	3107.435	0
2473.011	2	2932.4	0, uR	3109.276	2
2476.509	10, r	2936.570	0	3114.157	5, r
2482.05	0, U	2936.901	2	3122.917	0
2486.618	1	2938.552	0	3138.417	0
2489.010	4	2950.920	1	3139.531	0
2498.873	3	2951.134	0	3139.804	2
2503.597	0	2956.811	0	3142.932	6
2505.804	2	2962.443	0	3146.075	1
2513.0	1, U	2968.356	0	3147.730	0
2521.102	0	2975.953	0	3148.532	0
2536.872	2	2995.400	0	3168.022	1, u
2539.690	0	2996.660	0	3213.018	0
2544.821	4	3002.775	4, r	3219.088	4
2546.283	0	3009.903	3	3242.824	10, r
2546.990	0	3010.980	1	3251.754	5, r
2548.165	0	3014.733	1	3258.907	6, r
2551.095	0	3015.052	0	3272.925	2
2551.971	0	3020.835	2	3284.080	0, u
2564.0	2, UR	3021.859	3	3286.337	1
2565.595	0	3022.744	0	3287.378	5

PALLADIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3299.875	1, u	4087.518	6	5110.940	6
3300.325	0	4098. bis 4101.	, U	5114.530	2
3302.253	6, r			5117.158	7
3310.251	1	4123.761	2	5127.849	2
3311.136	2	4170.005	5	5161.491	1
3313.093	2	4213.116	6, r	5163.970	10
3317.455	0, u	4268.4	4, U	5209.044	4
3321.100	2	4321.8	0, U	5234.992	7
3346.268	0	4328.125	0, u	5256.321	3
3373.137	6, r	4344.8	4, U	5294.267	2
3380.840	5, u	4351.1	2, U	5295.744	10
3383.025	4	4358.773	0, u	5312.752	4
3388.811	0	4360.4	0, U	5345.278	4
3389.192	2	4379.8	0, U	5346.980	0
3396.081	0	4386.614	1	5362.864	4, uR
3396.926	3	4388.776	2	5363.474	1
3404.732	10, r	4406.759	5	5377.833	1, u
3406.210	1	4421.217	1, u	5385.668	0
3419.818	4	4443.191	3	5394.958	4, uV
3421.368	8, r	4458.785	2	5395.471	8, uR
3433.582	5, r	4469.307	0	5427.425	1
3441.548	6, r	4473.771	7	5435.379	3
3442.545	2	4489.641	4, u	5497.056	4
3460.888	7, r	4497.813	2	5529.657	6
3481.308	7, r	4516.406	5, u	5542.997	10
3488.293	0	4541.314	5, u	5547.219	9
3489.930	4, r	4553.096	2	5548.514	2, u
3517.087	8, r	4590.191	3, u	5562.902	2, u
3528.881	2	4632.770	2	5601.867	3
3553.242	7, r	4677.617	4	5608.229	5, u
3566.775	2	4708.261	0	5619.667	9
3571.305	5, r	4724.204	3	5621.520	2
3574.040	2	4762.098	0	5642.898	5
3596.795	2	4776.715	1	5655.628	5, u
3609.698	9, r	4788.327	8	5664.578	1
3634.840	10, r	4791.061	0	5668.605	2
3646.116	1	4817.26	0, u	5670.263	10
3654.574	2	4817.662	9	5674.432	2
3690.491	6, r	4822.347	0	5680.993	0
3719.061	4, r	4836.654	0	5687.670	2
3799.332	5, r	4875.577	7	5690.333	4
3894.335	6, r	4919.008	3	5695.293	9
3958.777	5, r	4924.373	0	5700.978	0
3992.5	1, U	4930.145	1	5736.826	5
4007.6	0, U	4972.081	3	5737.842	3
4011.8	0, U	5063.549	4	5739.881	4
4020.3	1, U	5101.704	1	5760.122	1
4021.2	0, U				

III. RUTHENIUM.

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2335.047	2	2471.576	0	2524.952	0
2338.094	2	2472.215	0	2525.263	0
2340.767	2	2474.115	1	2525.726	0
2342.920	2	2474.506	0	2526.011	0
2351.411	2	2475.483	2	2526.914	2
2357.991	2	2476.395	0	2528.027	0
2370.251	2	2476.960	2	2528.813	0
2375.346	2	2479.010	2	2529.812	1
2392.501	2	2479.458	0	2532.128	1
2396.791	2	2479.611	0	2533.331	1
2402.802	4	2481.216	0	2535.147	0
2407.997	2	2482.628	0	2536.315	0
2408.744	1	2484.055	0	2537.776	0
2420.905	2	2490.017	2	2538.565	0
2429.672	2	2490.555	0	2539.822	1
2434.980	0	2491.847	2	2540.411	0
2437.019	0	2494.116	2	2541.381	0
2439.715	0	2494.773	1	2542.601	0
2441.051	1	2495.775	2	2543.240	0
2441.419	0	2498.512	2	2543.349	2
2443.036	0	2498.670	2	2543.778	1
2444.129	1	2499.873	2	2544.318	2
2444.497	0	2500.484	0	2545.866	0
2444.924	0	2500.940	0	2546.765	2
2445.519	0	2501.569	2	2547.600	1
2447.537	1, u	2501.990	0	2549.260	0
2448.958	0	2502.484	0	2549.576	2
2449.958	1	2502.966	0	2549.664	2
2450.464	0	2507.090	2	2550.946	1
2450.650	1	2508.377	2	2551.466	1
2454.267	0	2508.508	2	2551.822	0
2455.005	2	2509.160	1	2552.083	0
2455.614	5	2509.709	0	2552.384	0
2456.376	0	2510.238	0	2552.524	0
2456.519	4	2511.058	0	2552.965	0
2456.666	4	2511.652	1	2554.060	1
2457.050	1	2512.898	2	2554.790	1
2457.311	0	2513.417	2	2555.734	0
2458.706	2	2515 372	1	2555.955	2
2459.146	0	2516.882	0	2556.100	2
2461.506	0	2517.403	2	2556.994	0
2463.026	2	2517.728	2	2557.784	1
2464.474	0	2518.601	0	2558.359	0
2464.781	2	2520.041	0	2558.626	2
2467.674	0	2520.925	1	2559.497	0
2470.608	0	2521.700	2	2560.347	2
2470.805	0	2522.410	0	2560.920	3

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2562.252	2	2609.143	4	2650.076	0
2564.503	0	2609.573	2	2650.486	1
2564.674	1	2611.130	2	2650.693	0
2565.277	1	2612.165	2	2650.968	0
2566.666	2	2612.990	0	2651.366	2
2567.981	1	2613.143	0	2651.603	0
2568.854	4	2614.151	1	2651.936	4
2569.840	2	2614.671	2	2652.240	0
2570.180	0	2615.179	2	2653.240	1
2571.068	2	2617.882	1	2653.776	1
2572.370	2	2619.105	0	2654.563	0
2572.512	2	2619.745	2	2654.898	0
2573.654	0	2620.154	0	2655.193	0
2575.339	1	2620.713	2	2655.292	1
2577.052	0	2621.173	0	2656.328	1
2578.653	2	2623.914	1	2656.641	1
2579.071	2	2625.168	0	2656.776	1
2579.309	1	2626.290	0	2657.249	1
2579.623	2	2626.444	0	2658.482	2
2579.879	0	2627.737	1	2658.862	0
2580.316	0	2628.375	4	2660.673	0
2580.883	2	2628.621	0	2661.249	2
2581.216	2	2630.010	0	2661.690	4
2581.990	2	2630.314	1	2661.937	0
2583.131	2	2631.657	1	2664.833	4
2584.211	2	2632.210	0	2665.227	0
2585.412	1	2632.584	1	2665.542	0
2585.815	0	2633.537	0	2665.803	1
2586.157	0	2635.451	0	2667.479	1
2587.413	0	2635.927	4	2668.042	1
2589.129	0	2636.617	0	2668.421	0
2589.649	2	2636.760	2	2670.586	0
2589.886	0	2638.597	2	2670.813	0
2591.087	1	2639.205	2	2672.451	0
2591.201	2	2640.413	2	2673.089	0
2591.710	0	2641.549	0	2673.550	2
2592.093	2	2642.063	0	2673.691	2
2594.926	2	2642.607	0	2674.930	0
2595.734	0	2643.042	4	2675.273	0
2596.043	0	2643.600	0	2676.430	2
2597.417	1	2644.187	0	2677.057	0
2598.681	0	2644.711	0	2677.406	0
2600.840	0	2646.087	2	2677.967	0
2601.392	0	2646.715	0	2678.267	0
2601.553	2	2647.394	2	2678.837	4
2604.409	0	2648.019	1	2679.843	1
2605.439	2	2648.535	1	2683.756	1
2605.950	2	2648.706	0	2684.172	1
2607.440	0	2648.872	2	2684.540	0
2608.024	1	2649.608	2	2685.242	0

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2686.375	4	2730.115	0	2782.305	1
2687.214	1	2730.416	2	2784.625	0
2687.580	1	2731.028	2	2784.978	0
2688.216	1	2732.011	0	2785.746	1
2688.668	1	2733.167	0	2787.930	3
2688.969	1	2734.438	3	2789.720	0
2690.487	1	2735.806	2	2790.695	0
2690.904	0	2736.412	0	2791.164	0
2691.199	4	2736.917	0	2792.418	2
2693.392	2	2738.983	0	2792.746	2
2693.750	0	2739.311	4	2795.464	0
2696.653	0	2740.085	0	2796.652	0
2697.595	0	2740.327	1	2800.243	0
2698.161	0	2744.022	2	2800.785	1
2699.957	1	2744.541	2	2802.260	0
2700.578	1	2744.821	0	2802.907	2
2700.772	0	2745.343	0	2803.593	1
2701.434	4	2746.169	0	2806.845	0
2702.916	4	2746.991	0	2808.335	0
2703.221	0	2749.923	0	2810.131	4
2703.403	0	2750.452	0	2810.645	3
2703.891	2	2751.698	0	2810.788	0
2705.416	0	2752.548	2	2811.360	0
2708.054	2	2752.868	2	2812.925	2
2708.930	0	2753.543	2	2813.807	0
2709.157	0	2757.175	0	2815.410	0
2709.291	2	2757.912	1	2817.192	3
2709.851	0	2758.104	0	2818.460	4
2710.321	0	2760.268	0	2818.913	0
2712.169	0	2762.400	2	2819.062	2
2712.493	4	2763.232	2	2819.667	0
2712.967	0	2763.513	4	2821.279	1
2713.272	2	2764.005	1	2821.504	0
2713.824	1	2764.824	2	2822.142	2
2715.326	0	2765.530	2	2822.371	0
2715.595	2	2766.323	0	2822.659	2
2717.100	0	2768.032	0	2822.912	0
2717.510	2	2769.024	4	2824.004	0
2718.919	0	2769 993	0	2824.866	0
2719.610	5	2770.399	0	2827.627	0
2719.838	0	2770.805	2	2827.969	4
2721.653	3	2772.716	0	2829.253	2
2721.937	0	2773.068	0	2830.815	1
2722.493	0	2774.589	2	2831.280	0
2722.760	3	2775.288	1	2832.755	0
2722.903	0	2775.723	0	2834.107	3
2724.153	2	2776.009	1	2836.254	2
2725.549	4	2777.629	0	2836.684	2
2727.063	0	2779.081	0	2837.384	0
2729.540	2	2780.858	2	2838.729	2

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2840.657	2	2888.112	2	2934.300	2
2841.777	2	2888.739	2	2934.638	0
2842.651	1	2889.543	0	2936.131	2
2842.859	0	2891.242	2	2936.380	0
2843.277	2	2891.762	2	2936.591	0
2846.430	1	2892.654	4	2937.448	1
2846.662	0	2893.844	0	2937.679	0
2848.688	1	2895.554	0	2939.247	2
2849.399	0	2895.925	1	2939.796	0
2851.225	1	2896.638	3	2940.057	3
2853.433	0	2897.820	1	2940.474	3
2854.173	4	2898.650	3	2942.366	1
2854.465	0	2898.845	1	2942.823	0
2854.820	0	2899.817	1	2943.593	1
2855.454	0	2901.890	1	2944.035	3
2855.995	2	2902.223	1	2944.294	0
2856.153	2	2902.969	1	2845.591	0
2857.367	1	2903.180	2	2945.775	4
2857.770	0	2904.825	0	2946.670	0
2858.693	0	2905.756	3	2947.102	4
2860.114	4	2905.952	1	2949.612	4
2860.491	0	2906.424	3	2950.080	0
2861.508	5	2908.590	0	2950.650	1
2861.833	1	2909.352	1	2951.516	2
2862.963	0	2910.542	2	2952.599	2
2863.112	2	2912.451	0	2953.116	0
2864.726	0	2912.555	0	2954.371	0
2866.743	5	2912.866	0	2954.594	4
2868.286	2	2913.286	3	2955.463	2
2868.426	2	2914.403	2	2955.714	0
2868.662	0	2915.736	2	2955.960	0
2869.047	0	2916.351	6	2957.297	0
2870.322	2	2917.249	2	2958.118	3
2871.296	3	2917.353	0	2958.993	0
2871.756	4	2917.880	2	2959.855	2
2872.468	2	2919.276	0	2961.097	2
2874.161	2	2919.723	4	2961.803	3
2875.104	5, u	2920.369	1	2962.442	0
2877.197	2	2921.068	2	2962.705	0
2877.930	2	2921.276	0	2963.523	2
2879.466	0	2924.760	0	2963.829	3
2879.853	3	2925.189	0	2964.415	0
2880.637	0	2925.685	0	2965.286	4
2881.373	1	2925.890	0	2965.670	3
2882.222	2	2926.913	0	2965.820	1
2882.697	2	2927.232	2	2966.674	1
2883.701	3	2927.858	0	2967.456	2
2884.601	2	2928.608	2	2968.233	0
2886.640	4	2929.027	0	2968.564	4
2887.224	0	2933.367	0	2969.069	4

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
2969.850	0	3018.158	2	3057.468	3
2972.594	0	3019.472	2	3058.762	2
2973.743	0	3019.876	0	3058.909	1
2974.099	3	3020.360	0	3059.284	3
2974.454	2	3020.989	2	3060.346	0
2975.253	1	3025.212	0	3062.155	2
2976.707	4	3027.195	2	3064.958	4
2977.048	3	3027.361	0	3068.355	4
2977.346	2	3027.678	0	3069.289	2
2977.596	0	3027.910	0	3071.586	2
2978.760	2	3028.785	0	3071.721	0
2979.847	3	3030.801	2	3071.824	0
2980.065	3	3030.890	2	3073.440	4
2981.080	0	3032.026	2	3075.412	1
2982.045	4	3032.771	0	3076.886	2
2986.104	0	3033.562	4	3077.175	2
2986.453	1	3034.167	4	3077.657	2
2988.047	1	3035.578	3	3078.209	1
2988.224	1	3036.580	1	3079.953	0
2989.079	6	3037.845	2	3080.292	4
2989.451	2	3038.078	2	3081.009	4
2989.770	2	3038.289	2	3081.218	1
2990.413	2	3038.851	0	3081.489	0
2992.080	0	3039.586	0	3081.946	0
2993.070	1	3040.071	2	3083.252	3
2993.387	3	3040.418	3	3084.631	2
2995.083	5	3042.025	1	3084.728	0
2997.011	3	3042.598	3	3085.597	0
2997.743	2	3042.953	2	3086.181	4
2998.446	3	3043.161	1	3086.631	2
2999.011	1	3044.077	0	3086.888	1
3000.341	2	3045.630	0	3087.039	2
3001.756	3	3045.833	4	3088.050	0
3002.188	0	3046.114	2	3088.177	2
3003.600	2	3046.356	2	3088.362	0
3004.708	2	3047.108	0	3089.252	4
3006.094	2	3048.442	0	3089.915	4
3006.708	4	3048.606	4	3090.341	2
3008.387	2	3048.897	4	3091.004	2
3008.695	0	3049.174	0	3091.974	2
3008.911	2	3050.309	1	3092.085	0
3009.798	0	3050.504	0	3092.351	0
3010.623	2	3051.704	2	3094.500	2
3012.003	0	3051.974	0	3095.640	0
3013.040	3	3052.445	1	3096.062	0
3013.172	0	3053.450	0	3096.672	6
3013.477	3	3055.042	4	3097.706	4
3014.312	0	3056.192	4	3098.954	0
3016.818	0	3056.877	0	3099.390	5
3017.356	5	3056.971	0	3100.953	4

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3104.070	0	3140.201	1	3189.835	3
3104.570	2	3140.596	3	3190.088	4
3105.382	2	3141.081	4	3191.303	1
3105.524	2	3143.764	0	3191.900	2
3105.910	0	3144.369	4	3192.171	2
3106.942	3	3144.820	2	3193.617	2
3107.373	0	3146.183	2	3195.137	0
3107.698	0	3147.323	2	3195.438	1
3107.829	3	3147.547	0	3196.718	4
3108.526	2	3148.138	0	3197.603	0
3110.147	0	3148.593	2	3198.437	2
3110.641	4	3150.283	1	3199.238	0
3112.012	3	3150.803	4	3201.372	2
3112.408	2	3151.780	1	3201.604	3
3112.782	2	3153.927	4	3202.703	2
3113.502	2	3154.543	2	3205.428	2
3113.756	0	3156.733	0	3207.751	0
3115.536	0	3156.917	2	3208.405	0
3116.945	1	3157.739	2	3208.542	3
3117.181	0	3159.003	4, Ca?	3208.865	1
3117.563	0	3160.036	4	3209.758	1
3118.170	4	3163.186	0	3210.287	2
3118.792	4	3164.939	0	3213.098	3
3120.650	1	3165.086	0	3214.475	2
3122.108	1 d?	3165.307	1	3215.613	0
3122.970	0	3165.507	0	3216.641	4
3123.610	0	3167.514	0	3219.274	1
3124.277	4	3168.355	1	3220.195	1
3124.481	2	3168.648	5	3220.899	2
3124.709	2	3170.196	2	3221.303	2
3126.068	4	3171.352	2	3221.493	1
3126.730	2	3172.778	0	3223.393	4
3127.387	1	3173.221	2	3223.723	0
3127.643	0	3173.500	2	3224.772	2
3128.539	2	3174.243	4	3225.418	0
3129.574	0	3176.401	3	3226.497	5
3129.717	2	3177.159	4	3227.016	2
3129.935	3	3178.843	1	3228.021	3
3130.709	0	3179.380	2	3228.276	2
3132.122	1	3180.569	0	3228.651	4
3132.988	4	3181.126	0	3228.850	0
3133.800	2	3181.312	0	3229.881	2
3134.895	1	3185.276	0	3230.738	2
3135.170	0	3185.553	2	3231.869	0
3136.044	2	3186.171	4	3232.180	1
3136.451	1	3186.867	1	3232.881	4
3136.663	3	3188.057	2	3233.650	0
3137.036	0	3188.463	5	3234.920	2
3138.884	2	3188.713	2	3235.230	2
3139.379	2	3189.418	2	3235.431	0

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3236.101	2	3272.366	0	3316.523	6
3238.132	0	3273.217	5	3317.045	1
3238.667	5	3273.765	0	3318.012	4
3238.904	2	3274.831	5	3318.992	6
3239.745	3, d	3276.820	0	3319.655	1
3241.362	4	3277.699	4	3319.944	1
3241.643	0	3279.521	2	3321.385	2
3241.884	0	3280.599	2	3321.634	0
3242.283	2	3280.678	1	3322.368	2
3242.978	2	3281.735	0	3323.226	4
3243.638	2	3281.995	2	3324.077	2
3244.475	0	3282.744	0	3324.509	0
3244.585	1	3285.067	4	3325.136	4
3244.719	0	3285.505	2	3325.373	2
3245.746	0	3286.040	1	3327.831	4
3246.380	0	3289.389	2, u	3328.583	2
3247.501	0	3291.250	2	3332.186	4
3248.977	2	3291.789	2	3332.483	0
3250.065	2	3292.390	2	3332.768	2
3250.146	2, Rh	3294.269	6	3334.764	0
3250.605	1	3294.926	0	3335.822	4
3251.464	3	3296.252	4	3336.296	2
3252.031	3	3296.786	2	3336.774	3
3252.400	0	3297.393	3	3337.963	4
3252.683	2	3298.096	3	3338.849	2
3253.038	1	3298.559	4	3339.092	0
3253.136	2	3299.479	2	3339.691	6
3254.674	4	3299.926	0	3339.932	2
3254.856	4	3301.726	5	3341.230	2
3255.173	0	3302.312	1	3341.361	1
3255.356	1	3304.141	4	3341.809	4
3256.477	4	3304.418	0	3342.854	0
3256.746	0	3304.634	2	3342.999	0
3258.176	3	3304.772	0	3344.666	4
3259.111	0	3304.948	2	3344.934	2
3259.811	4	3305.804	0	3345.450	4
3260.304	2	3306.305	4	3346.360	0
3260.494	5	3307.679	2	3347.748	4
3261.257	3	3308.122	4	3348.145	2
3263.740	0	3308.751	0	3348.833	2
3263.988	3	3309.965	0	3349.822	0
3264.692	3	3310.220	0	3350.236	2
3264.808	2	3311.090	4	3350.363	0
3266.588	4	3311.388	0	3350.681	2
3267.269	0	3312.068	0	3352.060	4
3268.345	5	3312.348	1	3353.122	1
3269.087	2	3314.203	2	3353.444	2
3269.336	2	3315.181	2	3353.776	4
3270.388	2	3315.365	3	3354.001	2
3271.746	0	3315.590	2	3355.803	2

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3356.327	2	3400.890	2	3444.574	1
3356.598	2	3401.304	0	3445.453	1
3358.110	0	3401.637	2	3445.675	0
3359.230	6	3401.878	3	3446.095	0
3361.295	2	3403.924	1	3446.227	2
3362.142	4	3405.426	0	3446.630	2
3362.457	2	3406.017	2	3449.105	4
3364.230	4	3406.736	2	3449.608	0
3364.933	1	3407.042	0	3451.014	0
3365.163	0	3409.420	5	3453.056	4
3365.470	0	3409.707	2	3453.373	0
3367.868	0	3411.768	4	3455.548	2
3368.053	0	3412.221	2	3455.888	2
3368.588	6	3412.947	3	3456.769	4
3369.433	2	3413.870	0	3457.849	0
3369.813	2	3414.130	0	3459.736	2
3370.720	2	3414.422	2	3462.208	2
3371.793	0	3414.787	3	3463.289	4
3371.990	4	3416.329	1	3463.751	0
3372.922	0	3417.493	7	3465.437	1
3374.115	2	3417.790	1	3467.190	2
3374.790	4	3418.125	2	3472.843	2
3375.036	2	3419.394	2	3473.900	5
3375.377	2	3420.243	4	3477.350	0
3376.186	1	3420.881	0	3480.295	2
3378.165	4	3422.578	2	3481.044	0
3379.402	2	3426.120	2	3481.465	4
3379.747	4	3427.717	0	3482.499	2
3380.301	4	3428.476	4, r	3483.317	2
3381.040	2	3428.790	2	3483.463	2
3383.053	0	3429.702	4	3486.360	2
3385.303	4	3430.568	0	3486.948	2
3385.609	2	3430.910	4	3489.895	1
3385.838	2	3431.905	0	3490.879	1
3386.390	2	3432.354	3	3492.256	1
3387.368	2	3432.560	0	3493.377	2
3387.967	0	3432.909	4	3494.410	3
3388.849	4	3433.406	4	3496.145	2
3389.250	0	3434.325	0	3496.293	2
3389.639	4	3435.340	4	3498.103	1
3391.042	2	3436.237	0	3499.098	10, r
3392.032	2	3436.481	2	3501.510	1
3392.654	4	3436.886	5, r	3502.578	2
3395.465	0	3438.522	4	3509.870	2
3396.060	0	3438.819	0	3513.807	2
3396.967	4	3439.835	2	3514.649	4
3398.470	0	3440.361	4	3514.911	1
3399.040	0	3441.942	0	3516.046	0
3400.116	0	3443.309	2	3519.795	3
3400.738	1	3443.818	0	3520.285	4

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3528.841	2	3637.614	4	3717.823	2
3531.545	3	3638.163	2	3719.474	4
3532.965	2	3640.791	4	3722.458	1
3535.529	2	3645.827	1	3724.110	4
3535.985	2	3646.266	3	3724.633	2
3538.100	3	3650.473	4	3725.115	4
3539.418	2	3652.465	3	3726.254	4
3539.518	2	3652.627	0	3727.077	4
3541.788	3	3652.816	0	3728.170	5
3547.136	1	3653.857	2	3730.587	7
3550.420	2	3654.559	4	3730.745	3
3554.002	1	3656.112	2	3731.045	2
3556.779	0	3657.315	2	3732.170	2
3557.203	0	3657.716	1	3733.187	2
3562.035	0	3660.964	3	3737.548	3
3564.517	0	3661.486	7	3737.904	2
3564.714	1	3661.727	2	3738.774	2
3564.945	0	3663.526	5	3739.058	2
3567.308	2	3668.890	1	3739.622	4
3570.743	2	3669.604	4	3742.435	5
3571.910	1	3671.363	2	3742.938	4
3574.744	3	3672.210	2	3744.367	2
3579.923	0	3672.525	2	3744.550	2
3587.344	2	3675.408	3	3746.372	2
3989.370	4	3676.817	3	3753.695	4
3591.044	1	3677.100	2	3755.241	3
3593.177	4, r	3678.140	2	3755.865	2
3596.315	5, r	3678.222	2	3756.083	4
3599.548	0	3678.465	4	3759.976	2
3599.913	4	3683.730	1	3760.178	4
3601.627	2	3685.204	2	3761.644	4
3605.792	3	3686.109	4	3764.179	1
3606.297	1	3686.742	1	3765.938	0
3608.862	2	3690.179	1	3767.500	4
3609.241	2	3693.740	2	3771.244	0
3614.486	1	3696.738	4	3773.306	0
3017.090	4	3697.921	3	3777.723	3
3619.334	4	3698.016	2	3781.313	3
3620.426	4	3700.487	1	3782.891	0
3623.804	4	3701.134	2	3786.193	5
5623.995	0	3701.457	2	3790.649	5
3625.345	5	3702.369	2	3795.327	0
3626.897	5	3703.344	2	3798.205	1
3627.425	2	3705.506	2	3799.040	4
3629.352	1	3712.443	3	3799.486	4, r
3631.860	3	3714.788	1	3812.874	3
3632.545	1	3715.703	3	3817.439	1
3934.063	4	3716.323	3	3819.184	2
3635.093	7	3716.583	1	3822.225	1
3635.661	4	3717.152	4	3825.075	1

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
3828.859	0	3965.057	4	4068.529	4
3831.946	4	3969.936	2	4071.560	3
3838.215	0	3972.568	0	4073.147	2
3839.815	1	3974.646	4	4073.260	2
3857.689	1	3978.620	5	4076.900	5
3867.965	1	3979.591	4	4079.440	1
3872.386	1	3982.043	2	4080.777	7
3884.203	2	3982.372	3	4082.947	2
3884.849	2	3984.840	1	4085.567	5
3887.962	2	3985.011	5	4091.218	1
3890.350	4	3987.959	4	4097.185	2
3891.567	2	3989.344	2	4097.965	4
3892.366	4	3994.700	1	4100.533	2
3892.916	2	3996.136	4	4101.906	4
3894.387	2	3996.650	2	4102.438	2
3897.390	2	4005.789	4	4106.065	0
3898.500	3	4006.749	4	4108.003	4
3901.393	4	4007.680	3	4108.218	2
3906.141	1	4008.422	2	4109.796	0
3908.907	3	4011.882	2	4112.910	4
3909.229	5	4013.655	4	4113.532	2
3911.279	3	4013.871	2	4114.285	1
3912.248	3	4014.297	2	4118.678	2
3915.000	4	4018.891	1	4121.147	2
3919.711	0	4019.699	2	4121.287	2
3921.060	4	4021.146	3	4123.227	2
3922.476	1	4022.327	5	4127.611	2
3923.636	6	4022.837	2	4128.017	2
3924 776	2	4024.001	4	4137.410	3
3926.071	6	4024.449	2	4138.923	0
3926.581	0	4024.848	2	4144.335	4
3931.936	4	4026.650	1	4145.905	4
3932.444	0	4028.584	2	4146.956	4
3934.352	1	4031.147	3	4148.530	1
3938.045	3	4032.363	4	4150.475	1
3939.268	0	4032.650	1, u	4161.817	4
3941.811	3	4036.612	2	4167.030	0
3942.209	4	4037.892	2	4167.666	5
3944.341	2	4039.370	4	4170.218	2
3945.723	0	4040.620	2	4175.615	2
3946.456	2	4042.123	2	4182.621	2
3949.564	2	4049.570	2	4182.807	1
3950.192	2	4051.566	4	4182.994	0
3950.366	4	4052.356	4	4189.639	0
3950.548	3	4054.216	4	4197.038	2
3951.351	4	4063.021	1	4197.748	4
3952.436	1	4063.160	2	4199.039	4
3952.850	5	4064.262	2	4200.069	7
3957.376	0	4064.616	4	4206.178	4
3957.696	2	4067.777	4	4207.797	2

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
4212.240	5	4315.219	4	4396.868	0
4214.610	4	4316.792	2	4397.956	4
4217.438	7	4318.596	4	4399.751	1
4220.838	4	4319.274	2	4405.809	0
4225.258	3	4320.045	5	4410.207	6
4226.825	0	4320.743	2	4412.058	0
4229.472	4	4320.972	0	4413.458	2
4230.470	6	4321.450	2	4414.607	1
4232.478	4	4323 120	2	4420.634	2
4236.838	4	4323.626	0	4421.006	4
4240.194	0	4325.215	4	4421.629	4
4241.231	6	4326.987	4	4423.143	1
4243.228	6	4327.489	2	4424.958	3
4244.997	4	4327.588	3	4426.182	1
4246.359	0	4328.712	2	4428.624	4
4246.522	4	4331.321	4	4430.478	1
4246.902	4	4332.655	2	4439.574	2
4248.304	2	4332.789	0	4439.938	5
4255.868	1	4336.584	2	4440.245	0
4256.049	0	4337.427	4	4444.674	4
4256.790	0	4338.829	2	4449.509	4
4259.152	5	4340.503	2	4460.209	6
4260.166	3	4341.204	2	4464.661	0
4263.551	2	4342.243	6	4465.649	1
4265.766	2	4343.178	0	4466.511	2
4266.157	0	4346.640	4	4467.427	2
4273.115	0	4349.868	5	4471.200	0
4277.415	2	4350.632	0	4474.093	4
4278.842	2	4354.300	6	4475.493	2
4282.093	2	4354.960	3	4480.603	4
4282.357	2	4357.031	1	4482.194	2
4284.502	6	4361.372	5	4488.550	4
4287.209	4	4361.581	2	4490.396	2
4290.692	2	4362.872	1	4491.846	2
4292.419	0	4364.270	2	4498.322	4
4293.441	4	4365.741	0	4508.192	1
4294.268	4	4370.580	2	4508.715	2
4294.955	5	4371.363	4	4510.251	4
4296.090	5	4372.381	5	4511.353	4
4296.860	2	4376.745	1	4516.421	2
4297.887	8	4381.421	2	4517.060	4
4301.297	1	4383.530	2	4517.977	4
4302.150	0	4385.563	5	4521.110	4
4307.748	4	4385.823	5	4525.616	0
4308.567	0	4386.431	4	4531.035	4
4309.361	2	4389.150	2	4542.848	1
4312.047	0	4389.547	0	4547.105	2
4312.632	2	4390.614	6	4547.463	4
4313.067	0	4391.191	4	4548.030	4
4314.468	4	4395.125	2	4549.589	2

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
4550.112	3	4716.201	2	4902.033	0
4552.281	4	4718.228	0	4903.223	5
4554.696	6, r	4721.078	1	4905.179	1
4559.215	1	4731.504	3	4908.045	3
4560.157	4	4733.486	0	4910.384	0
4562.772	1	4733.710	4	4911.755	1
4564.862	2	4738.587	0	4921.233	4
4580.246	3	4743.205	1	4935.805	0
4584.632	4	4751.197	0	4938.587	3
4589.177	0	4753.280	0	4955.416	1
4589.734	0	4756.402	2	4960.022	0
4591.257	4	4758.043	6	4969.055	2
4591.717	2	4764.582	0	4974.255	0
4592.695	4	4767.315	0	4975.534	0
4593.161	0	4769.464	4	4976.351	2
4593.367	0	4773.325	0	4980.498	2
4596.879	4	4774.168	0	4987.412	1
4599.271	6	4781.937	1	4992.891	2
4601.933	3	4794.547	2	5003.697	0
4602.978	0	4795.721	2	5005.394	1
4605.833	2	4798.607	2	5010.765	1
4617.827	0	4801.343	1	5011.387	3
4626.184	1	4805.043	2	5019.140	1
4628.495	0	4806.375	0	5020.472	0
4635.849	4	4813.412	0	5026.343	3
4638.569	0	4814.895	0	5039.794	0, u
4639.490	0	4815.694	5	5040.521	1
4641.135	0	4817.512	1	5040.908	1
4642.548	1	4822.738	0	5041.528	0
4642.752	1	4828.865	0	5045.570	1
4645.264	4	4833.157	2	5047.471	2
4646.326	0	4839.174	3	5053.114	0
4646.967	0	4839.930	1	5057.487	4
4647.787	5	4844.720	4	5062.815	1
4648.293	0	4854.731	1	5073.141	2
4652.371	0	4862.024	2	5077.243	1
4654.489	4	4863.265	0	5077.484	3
4654.901	0	4865.253	1	5093.996	4
4662.663	0	4869.314	6	5101.553	2
4670.146	4	4869.952	1	5101.892	0
4674.821	4	4874.489	0	5107.230	4
4681.563	0	4875.188	0	5127.423	2
4681.966	4	4877.598	0	5134.059	2
4683.258	0	4882.832	0	5134.285	0
4684.196	4	4885.186	0	5136.717	5
4685.947	1	4895.474	1	5142.933	4
4690.284	4	4895.555	1	5147.401	4
4709.672	6	4895.745	4	5151.230	4
4712.146	1	4899.416	1	5153.364	2
4714.335	0	4901.234	0	5155.302	4

RUTHENIUM—*continued.*

Wave-length	Intensity and Character	Wave-length	Intensity and Character	Wave-length	Intensity and Character
5160.167	2	5419.056	4	5653.005	0
5168.237	0	5427.815	4	5653.482	2
5168.793	0	5439.421	2	5657.127	2
5169.242	0	5439.618	2, u	5663.233	0
5171.193	6	5452.930	1	5665.370	4
5174.105	0	5455.018	6	5676.720	1
5176.361	0	5456.329	2, u	5679.790	4
5195.171	4	5471.755	0	5688.990	2
5200.040	3	5473.050	2	5692.288	1
5202.285	2	5475.377	2	5693.190	4
5209.667	2	5479.619	4	5694.626	2
5213.586	3	5480.507	3	5696.526	1
5214.247	1	5484.524	6	5699.224	9
5223.708	3	5484.850	2	5699.741	2
5235.774	1	5494.575	1	5702.522	4
5242.560	1	5596.899	4	5713.025	4
5243.109	2, u	5501.230	1	5714.391	2
5245.112	0	5507.151	0	5724.975	4
5245.612	2	5510.934	6	5725.895	4
5251.816	1	5512.593	2	5730.122	2
5257.240	2	5518.056	2	5734.606	0
5264.113	0, u	5531.220	2	5740.710	0
5266.642	1	5540.881	3	5745.776	1
5266.988	1	5549.960	2	5746.131	4
5275.240	1	5556.719	3	5747.623	5
5280.989	2	5559.962	6	5752.163	3
5284.256	4	5569.233	4	5753.772	1
5291.327	1	5570.906	2	5756.980	3
5305.030	4	5578.594	4	5758.875	0
5306.035	0	5578.914	2	5768.066	3
5306.624	1	5579.650	2	5771.352	0
5307.481	0	5582.501	2	5774.533	2
5309.440	4	5600.753	2	5782.511	2
5315.520	2	5603.370	2	5782.720	4
5333.114	3	5603.782	3	5790.741	1
5334.901	2, u	5606.958	3	5792.382	1
5336.110	3	5609.360	2	5804.461	4
5348.340	0	5619.558	0	5815.157	5
5361.967	5	5627.722	2	5826.018	0
5362.271	2	5629.984	1	5828.235	2
5365.799	2	5636.441	7	5828.580	1
5373.505	0, u	5641.848	2	5833.380	2
5378.042	3	5647.755	0	5833.561	0, u
5386.083	4	5648.058	1	5864.830	0
5401.234	5	5649.737	3	5887.371	0
5401.609	2	5650.981	2		

THE ABERRATION OF PARABOLIC MIRRORS.

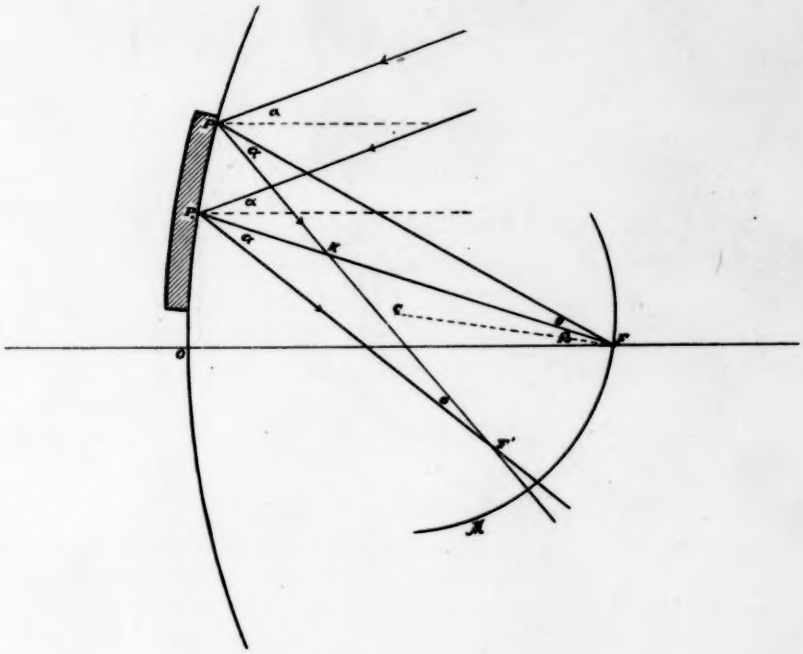
By CHARLES LANE POOR.

FOR some months I have been experimenting with parabolic mirrors for reflecting telescopes, being especially interested in attempts to grind and polish a new form of mirror, which promised, apparently, many distinct advantages over the ordinary form. These experiments led to an investigation of the theory of the optical properties of different portions of a paraboloid of revolution. To my knowledge no satisfactory investigation of these properties has been published. Astronomers and instrument makers have been satisfied with the fact that one portion of the field is theoretically perfect, and have assumed, apparently, that the field is sensibly flat and that the aberrations in different portions are insensible. These latter are far from being the facts, as is admirably pointed out in the paper by Professor Schaeberle in *Astronomical Journal*, No. 413.[1] The formulæ contained in his paper are approximate and are derived in an approximate and unsatisfactory manner. I have, therefore, put my results in shape and give them in the following paragraphs. These formulæ are rigorously derived and are accurate. They give rise to several interesting theorems concerning the field of a reflector and the various foci of different portions of the mirror.

Let the following figure represent a section of a paraboloid, OF being the axis and F the principal focus. Further let P_o be the central point of a mirror formed from the paraboloid; β being the angular distance of the point, P_o, from the central point of the paraboloid: θ is the half angular aperture of the mirror.

[1] PROFESSOR SCHAEBERLE does not seem to recognize that he is dealing with a simple case of what has hitherto been known in the theories of both lenses and mirrors as "aberration." The word, aberration, does not appear in his paper. He introduces a new and doubtful term, the so called "*blurring factor.*"

Now put: f, the focal length of the parabola,
r_o, the radius vector drawn to P_o,
r, the radius vector drawn to P.



Suppose a beam of parallel light be incident upon the mirror and making an angle, α, with the axis of figure. The central ray of this beam is incident at P_o and is reflected in the direction P_oF' making an angle, α, with the radius vector FP_o. The ray incident at P is reflected in the direction PF', making an angle, α, with the radius vector PF and intersecting the first ray at F'. The distance P_oF' is the focal length for the ray under consideration; call this distance f'. This distance will evidently be a function of α, β, and θ; and a general expression for it is derived as follows:

The two triangles PKF and P_oKF' are similar, hence their corresponding sides are proportional, whence:

$$f' = r\frac{P_0K}{PK} \tag{1}$$

But in the triangle PKF we have:

$$PK = r\frac{\sin\theta}{\sin(\alpha+\theta)} \tag{2}$$

and

$$FK = r\frac{\sin\alpha}{\sin(\alpha+\theta)} \tag{3}$$

Again from the figure,

$$P_0K = r_0 - FK. \tag{4}$$

Substituting in equation (1) the values of PK and P_0K as found in (2), (3), and (4), we have:

$$f' = r_0\frac{\sin(\alpha+\theta)}{\sin\theta} - r\frac{\sin\alpha}{\sin\theta} \tag{5}$$

From the theory of the parabola we have:

$$r_0 = \frac{f}{\cos^2 \tfrac{1}{2}\beta} \tag{6}$$

$$r = \frac{f}{\cos^2 \tfrac{1}{2}(\beta+\theta)}$$

Substituting these values in equation (5) it at once takes the form:

$$f' = \frac{f}{\sin\theta\cos^2\tfrac{1}{2}\beta}\left[\sin(\alpha+\theta) - \sin\alpha\frac{\cos^2\tfrac{1}{2}\beta}{\cos^2\tfrac{1}{2}(\beta+\theta)}\right] \tag{7}$$

By writing,

$$\alpha = (\alpha+\theta) - \theta \tag{8}$$

and expanding the sine and substituting the result in (7), it may finally be put in the form:

$$f' = \frac{r_0}{2\cos^2\tfrac{1}{2}(\beta+\theta)}\left[\cos(\alpha+\theta) + \cos(\alpha+\beta+\theta) + \sin(\alpha+\theta)\tan\tfrac{1}{2}\theta\right] \tag{9}$$

And this is a rigorous expression for the focal length of oblique rays for a mirror cut from any portion of the paraboloid.

If in this equation we put α equal to zero, it reduces to,

$$f' = r_0$$

an expression independent of θ. This shows that when the incident beam is parallel to the axis of figure, the rays from all por-

tions of the parabola are brought rigorously to a single focus. This is, of course, the well-known property of the parabola.

If we put θ equal to zero our general equation becomes:

$$f_o' = \frac{r_o}{2\cos^2 \frac{1}{2}\beta}\left[\cos\alpha + \cos(\alpha+\beta)\right] \qquad (10)$$

That is, as the angular aperture of the mirror is reduced, the focal length for oblique rays tends to the above as a limit. The locus of f_o', as α takes different values, is, therefore, the field of the mirror. For a given mirror, or what is the same for a given value of β, the locus of f_o', as given by the above equation, is a circle. If for a moment, we consider P_o as origin and r_o as the axis of X, then the coördinates of the center of the above circle and its radius, ρ, are given by:

$$\begin{aligned} x &= \tfrac{1}{2} r_o \\ y &= -\tfrac{1}{2} r_o \tan \tfrac{1}{2}\beta \\ \rho &= \tfrac{1}{2} r_o \sec \tfrac{1}{2}\beta. \end{aligned} \qquad (11)$$

This circular field is shown on the diagram by the circle, FM, whose center is at C.

The field as actually used in any telescope is flat and is the plane tangent to the true field at the focus, F. A photographic plate is plane and is usually placed so that F is the center of the apparent field. The tangent plane at F is perpendicular to the radius CF, and, therefore, this plane is inclined at an angle, $90^\circ - \frac{1}{2}\beta$, to the ray of light incident at its center F. For small values of β this probably does no harm, but for large values of β it renders the mirror useless for many astronomical purposes.

From the figure, as the triangles PKF and P_oKF' are similar, the four points, P_o, P, F, and F' lie on the circumference of a circle. This gives the following theorem:

"For incident light, oblique to the axis of figure, the intersection of the central ray with the ray from any point, P, of the parabola, lies on the circumference of the circle passing through the center of the mirror, the principal focus, and the reflecting point, P, of the mirror."

ABERRATION.

The aberration in different portions of the field of the parabolic section may be found rigorously in the following manner:

For oblique light the linear aberration in the direction of the central ray will be:

$$\Delta f' = f' - f_0'.$$

Substituting in this the values of f' and f_0', as given by (9) and (10), we have at once:

$$\Delta f' = \frac{r_0}{2\cos^2 \tfrac{1}{2}(\beta+\theta)}\left[\cos(\alpha+\theta)+\cos(\alpha+\beta+\theta)+\sin(\alpha+\theta)\tan\tfrac{1}{2}\theta\right]$$
$$-\frac{r_0}{2\cos^2\tfrac{1}{2}\beta}\cdot\left[\cos\alpha+\cos(\alpha+\beta)\right].$$

Expanding the sines and cosines contained in the above parentheses, and simplifying the resulting expression, we shall finally obtain:

$$\Delta f' = -\frac{r_0 \sin\alpha \sin\theta\,[2+2\cos\beta+\cos\theta+\cos(\theta+\beta)]}{8\cos^2\tfrac{1}{2}\beta\,.\,\cos^2\tfrac{1}{2}\theta\,.\,\cos^2\tfrac{1}{2}(\beta+\theta)} \qquad (12)$$

And this is the rigorous expression for the linear aberration of an oblique ray. We note in the above that the parenthesis and the denominator are both functions of β and θ alone. Calling these P and D respectively, we may write:

$$\Delta f' = -r_0 \sin\alpha \sin\theta \frac{P}{D} \qquad (13)$$

Now let σ represent the transverse linear aberration in the circular field, and we shall have:

$$\sigma = \Delta f' \tan\theta.$$

The injurious effect of the aberration is proportional to the angle which σ subtends at the center of the mirror, and calling this angle γ, we have finally:

$$\tan\gamma = \frac{\sigma}{f_0'}$$
$$= \frac{\Delta f'}{f_0'}\tan\theta.$$

Substituting in this the values of $\Delta f'$ and f_o' and reducing the result, we have

$$\tan \gamma = \frac{\sin \alpha \tan \tfrac{1}{2}\theta \tan \theta}{2\left[\cos \alpha + \cos (\alpha + \beta)\right] \cos^2 \tfrac{1}{2}(\beta + \theta)} \cdot P. \qquad (14)$$

And this is an expression for the angular size of a star disk in any portion of the field.[1]

If in this expression we put α equal to zero, then we have at once γ equal to zero; that is, the image of a star in the direction of the axis of figure is a point. And this is true for a mirror cut from any portion of the paraboloid; it is the well-known property of the parabola.

The equations so far derived are perfectly general, applying to all portions of the paraboloid. The properties of different portions may now be investigated by giving β proper values.

THE ORDINARY FORM OF MIRROR: $\beta = 0$.

If in our general equations we put β equal to zero, then the center of the mirror will coincide with o, and the mirror will be symmetrical with respect to the axis of figure of the paraboloid. This is the form in which nearly all mirrors for astronomical work have been made. Equations (9), (10), and (14) respectively reduce to:

$$f' = f \sec^2 \tfrac{1}{2}\theta \left[\cos (\theta + \alpha) + \tfrac{1}{2} \tan \tfrac{1}{2}\theta \,.\, \sin (\theta + \alpha) \right]$$

$$f_o' = f \cos \alpha$$

$$\tan \gamma = \tan \alpha \tan \theta \tan \tfrac{1}{2}\theta \left[1 + \frac{1}{2 \cos^2 \tfrac{1}{2}\theta} \right].$$

The second of these equations shows that the field is circular, and that the diameter of the circle is the focal length of the mirror. The complete mirror is formed by revolving the sec-

[1] The above formula gives the angular diameter of the star image measured in the plane passing through the axis of the paraboloid. The image will not be circular, although for stars near the center the images will not differ greatly from circles. The equations show that the light will not be uniformly distributed over the image. Professor Wadsworth has kindly given me valuable suggestions on this point, and I hope to discuss this subject in a subsequent paper.

tion about the axis of figure; hence the entire field will be spherical with f as diameter. The field used for photography, or for visual purposes, is plane; hence, even if the star images in the true field be points of light, their projections on the plate or in the eyepiece will be disks. The images in the true field are disks, and these will be, therefore, enlarged before they reach the flat plate. In every case the curvature of the field increases the injurious effect of the aberration.

The expression for γ gives the greatest diameter of a star disk in any portion of the true field. A very convenient and close approximation to it may be made by putting

$$\cos \theta = \cos \tfrac{1}{2} \theta = 1 .$$

Doing this and also expressing the angle a in seconds of arc, we have

$$\gamma = \frac{3}{4} \frac{a^2}{f^2} (a'') \qquad (15)$$

where a is the half linear aperture of the mirror. The size of the star disk is thus directly proportional to its angular distance from the center of the field and to the square of the proportions of the telescope (aperture divided by focal length).

The annexed table shows the results as obtained above and applied to reflectors of different proportions. The first column gives the ratio of aperture to focal length; the next two columns the size of a star disk on the field of least aberration for various distances from the center of the field; the final column gives the real size of a star disk as it would appear on a photographic plate and distant one degree from the center. This column contains the combined effect of aberration and curvature of the field (see table on next page).

As a result of this we see that in order to obtain satisfactory results the proportions of mirrors should be about the same as those of lenses: the ratio of aperture to focal length should be 1 to 15 or 20. Good definition cannot be obtained with the mirrors now in use, *i. e.*, those in which this ratio is 1 to 7 or 8.

TABLE I.[1]

Aperture. Focal length	Aberration for 5′	Aberration for 30′	Star disk 1″ from center
1/ 5	2″.16	12″.95	33″.3
1/ 6	1 .54	9 .24	23 .9
1/ 7	1 .14	6 .82	18 .2
1/ 8	.87	5 .24	14 .4
1/10	.55	3 .30	9 .8
1/12	.39	2 .37	7 .3
1/15	.25	1 .48	5 .1
1/20	.14	.84	3 .2

UNSYMMETRICAL FORM: $\beta = 10°$ OR LESS.

This form has been suggested once or twice, and Dr. Draper constructed one or two such mirrors. These, however, were the results of accidents in grinding and polishing, and not the results of attempts to make such mirrors. In these cases β was about two degrees.

The general equations (9) (10) and (14) now apply and in these β must be given the proper value for each special case.

We note first that the field is circular and slightly flatter than in the ordinary form. But the field is now inclined, and to obtain the best results the plate should not be perpendicular to the central ray, but inclined to it at an angle of $90° - \frac{1}{2}\beta$. This inclination of the field will cause a distortion of the star images, but this effect will be very small for small values of β.

[1] To test the accuracy of these results a number of measures were made on a print of Roberts' photograph of the Orion nebula, taken December 18, 1886, with an exposure of fifteen (15) minutes. The star disks at various distances from the center were measured, stars of the 9.3 and 9.4 magnitudes alone being used. The images increase in size from center outward, and this increase is shown below, together with the approximate aberration as computed from my formula:

Distance from center of photograph	Measured increase in star image	Computed aberration
12′	6″	6″
27′	9″	12″
34′	13″	15″
42′	19″	19″

The amount of the aberration for mirrors of different proportions and for different values of β can be computed from equation (14). Results of such computations are contained in the annexed table. Mirrors of four different ratios are compared and the sizes of the star images computed for different values of β, from 0° to 10°. In each and every case the greatest diameter of the star disk 30′ from the center of the field is given: the first column, $\beta = 0$, gives the size of star disk in the ordinary form of mirrors.

TABLE II.

Aperture. Focal length	$\beta = 0°$	$\beta = 2°$	$\beta = 4°$	$\beta = 6°$	$\beta = 10°$
1/ 6	9″.24	9″.26	9″.28	9″.30	9″.37
1/ 8	5 .24	5 .25	5 .26	5 .27	5 .31
1/10	3 .30	3 .31	3 .31	3 .32	3 .34
1/20	.84	.84	.84	.85	.86

From this table we see that such unsymmetrical mirrors perform nearly as well as the ordinary form. Thus two mirrors whose apertures are one eighth their respective focal lengths, the first symmetrical and the second cut at a point 10° from the axis, will give star images of 5″.24 and 5″.31 respectively. If the proportions are made one to twenty, then the oblique mirror will perform as well as the mirror of the usual type.

Such an oblique mirror would offer several advantages. In the first place it could be made of much greater focal length than the ordinary form, and again it would do away with the troublesome diffraction caused by the supports of the secondary mirror, or the photographic plate, as used in the ordinary form.

NINETY-DEGREE FORM: $\beta = 90°$

This form was suggested some years ago by Professor Pickering. It offers many distinct advantages, especially in the resulting form of equatorial mounting; and this led me to an attempt to grind and polish a small mirror of this type. These experiments were described at the astronomical conferences

held in connection with the dedication of the Yerkes Observatory.

To investigate this form put β equal to 90° in the general equations and they will reduce to

$$f' = \frac{r_0}{1-\sin\theta}\left[\cos(\alpha+\theta)+\sin(\alpha+\theta)(\tan\tfrac{1}{2}\theta-1)\right]$$

$$f_0' = r_0(\cos\alpha-\sin\alpha)$$

$$\tan\gamma = \frac{\tan\theta}{1-\cos\alpha}\left[\tan\tfrac{1}{2}\theta+\tan\theta\tan(45^\circ+\tfrac{1}{2}\theta)\right].$$

From the second of these we see that the field is circular and that the coördinates of the center of the field are, $\frac{1}{2}r_0$ and $-\frac{1}{2}r_0$. The field is therefore inclined at an angle of 45° to the central ray and to the axis of figure. It is, however, flatter than the field of the usual form, the radii of the circles for similar mirrors of the two types being in the ratio of 1 to $1/\sqrt{2}$. The inclination of the field renders this proposed form of mirror useless for many astronomical purposes, for only one portion of the field can be brought into focus with the eyepiece at any one time; but for some astrophysical investigations it may prove of value.

Again, from our final equation we find that the aberrations of a mirror of this form are very much larger than those of the ordinary symmetrical form. For a mirror of ratio one to ten the aberration at a distance of 30′ from the center of the field will be 6″.9 as against 3″.3 for the ordinary form.

JOHNS HOPKINS UNIVERSITY,
November 30, 1897.

ON THE CAUSES OF THE SUN-SPOT PERIOD.

By E. J. Wilczynski.

I.

In my inaugural dissertation, "Hydrodynamische Untersuchungen mit Anwendungen auf die Theorie der Sonnenrotation," Berlin, 1897, I have already mentioned briefly a very simple mechanical explanation of the Sun-spot period.

If every point of the fluid matter of which the Sun consists describes a circle, and taking into account the internal fluid friction, the following equations were found :[1]

$$-\omega^2 r = f\frac{\delta V}{\delta r} - \frac{1}{\rho}\frac{\delta p}{\delta r}, \qquad o = f\frac{\delta V}{\delta z} - \frac{1}{\rho}\frac{\delta p}{\delta z} \tag{1}$$

$$\frac{\delta \omega}{\delta t} = \frac{k}{\rho}\left(\frac{\delta^2 \omega}{\delta r^2} + \frac{3}{r}\frac{\delta \omega}{\delta r}\right) + \frac{\delta k}{\delta r}\frac{\delta \omega}{\delta r}, \tag{2}$$

where the axis of z is the axis of rotation, and $r = \sqrt{x^2 + y^2}$ is the distance from the axis. ω, ρ, p, k, V, denote respectively the velocity of rotation, density, pressure, coefficient of friction, and potential for the point whose coördinates are x, y, z. f is a constant depending only upon the units employed. If the surfaces of constant density and those of constant pressure coincide ω is a function of r and t only.

Our general view of the case is simply this: ω can be any function of r which verifies the above equations. The special form of this function has been determined by the initial conditions in the original solar nebula, just as the elements of a planetary orbit were determined by the position and by the magnitude and direction of the planet's velocity at the time of its formation. That the whole Sun should rotate as a solid body is then just as improbable as that a planet's orbit should be an exact circle.

[1] "Hydrod. Unters.," p. 7, of this Journal, **4**, 101, August 1896, where equations (1) are also deduced. An error in this proof was corrected by Harzer, *A. N.*, 3386.

The tendency of friction is towards a rigid rotation of the entire body, *i. e.*, $\omega = \text{const.}$ But besides this secular change in ω there can be periodic changes, say with an eleven-year period. In order to demonstrate this we will not take the most general case, it being easy to see that our conclusions will admit of being applied to cases which are not covered by the rigid demonstration.

In equations (1) any function of r and t can be put for ω, so that these equations need not be investigated. In order to simplify equation (2) so as to enable us to study its solutions, we will put ρ and k equal to constants, corresponding to the case of an incompressible homogeneous fluid. Then (2) becomes

$$\frac{\delta \omega}{\delta t} = a^2 \left(\frac{\delta^2 \omega}{\delta r^2} + \frac{3}{r} \frac{\delta \omega}{\delta r} \right) \tag{3}$$

if

$$\frac{k}{\rho} = a^2. \tag{4}$$

Attempt to solve (3) by putting

$$\omega = e^{\phi(r)\, t + \psi(r)}$$

This will verify (3) if [1]

$$\phi(r) = a^2 \left[\phi''(r)\, t + \psi''(r) + \left\{ \phi'(r)\, t + \psi'(r) \right\}^2 + \frac{3}{r} \left\{ \phi'(r)\, t + \psi'(r) \right\} \right]$$

for all values of r and t. But this is only possible if the coefficients of like powers of t on the left and right hand member are equal. Hence

$$\begin{cases} \phi(r) = a^2 \left[\psi''(r) + \psi'(r)^2 + \frac{3}{r} \psi'(r) \right] \\ 0 = \phi''(r) + 2\, \phi'(r)\, \psi'(r) + \frac{3}{r} \phi'(r) \\ 0 = \phi'(r)^2. \end{cases}$$

The last two equations are satisfied for $\phi(r) = c$, where c is any real or complex constant. Then ψ is determined by the equation

[1] $\phi'(r)$, $\phi''(r)$, etc., denote the first and second derivatives as usual.

$$\frac{d^2\psi}{dr^2}+\left(\frac{d\psi}{dr}\right)^2+\frac{3}{r}\frac{d\psi}{dr}=\frac{c}{a^2}=q.$$

Put $\psi=\log u$, then

$$\frac{d^2u}{dr^2}+\frac{3}{r}\frac{du}{dr}-qu=0 \tag{5}$$

a well-known equation, closely related to Bessel's.

Now let c_k be any constant, let u_{k1}, u_{k2} be the independent solutions of (5), if instead of q we put $q_k=\frac{c_k}{a^2}$. Then the corresponding complete solution of (5) is

$$u_k=a_{k1}u_{k1}+a_{k2}u_{k2}$$

where a_{k1} and a_{k2} are arbitrary constants. A very general solution of our equation for ω will then be

$$\omega=\sum_{k=1}^{\infty}(a_{k1}u_{k1}+a_{k2}u_{k2})\,e^{c_kt}. \tag{6}$$

The real parts of c_k must not be positive since ω must be finite for $t=\infty$. If the real part is zero, the imaginary part must also be zero, since for $t=\infty$ we must assume ω to have assumed a value ω_∞, which is independent of t, and also of r, *i. e.*, it is the same all through the entire body, so that ω_∞ is an absolute constant. We can therefore put

$$\omega=\omega_\infty+\sum_{k=1}^{\infty}(a_{k1}u_{k1}+a_{k2}u_{k2})\,e^{c_kt} \tag{7}$$

where the real parts of the c_k's are all negative. If the quantity on the right member be a complex quantity, we must put ω equal to the real part. We will then have ω given as a series, all of whose terms have the following form :

$$Au\,e^{-c^2t}\cos\lambda t \quad \text{or} \quad Au\,e^{-c^2t}\sin\lambda t \tag{8}$$

where A, c, and λ are real constants, and u is a function of r verifying (5). The period of such a term is of course

$$\frac{2\pi}{\lambda}$$

after which time the term will again have its original value multiplied by

$$e^{-c^2 \frac{2\pi}{\lambda}}$$

which can be made as nearly equal to unity as may be required by simply choosing the arbitrary c small enough. Of course λ can be so chosen that the period is eleven years. But this will determine the value of q which occurs in (5). The solution of this equation gives u_1 and u_2, and thus will be obtained as a part of the expression for ω the real part of

$$(a_1 u_1 + a_2 u_2)\, e^{(-c^2 + \lambda\sqrt{-1})t}$$

If this does not represent the observations, another term must be taken, etc.

But far more important than this would be the solution of the following problem: For $t = 0$, let $\omega = \omega_0$ an arbitrary function of r, be given. Equation (3) being of the first order in t, ω is then determined for all values of t. Then, letting ω_0 represent the present law of solar rotation, the future variations of ω could be deduced and compared to the facts of solar physics, as for instance the eleven-year period of solar activity. Thus important conclusions could be drawn.

II.

We will not on this occasion enter upon the discussion of this problem. It suffices for our purpose to have shown the possibility that ω as a function of t may contain periodic terms. Now from (1) we have

$$\omega^2 = \frac{1}{r}\left(\frac{1}{\rho}\frac{\delta p}{\delta r} - f\frac{\delta V}{\delta r}\right)$$

and putting

$$p = c\rho T$$

according to the law of gases, where T denotes the absolute temperature in centigrade degrees, and where c is a constant, we obtain

$$\omega^2 = \frac{1}{r}\left[c\frac{\delta T}{\delta r} + c\frac{T}{\rho}\frac{\delta \rho}{\delta r} - f\frac{\delta V}{\delta r}\right]. \qquad (9)$$

Now, suppose ω^2 to be a periodic function, then the bracket on the right-hand member must also have the same period. We will, moreover, assume that the density remains comparatively constant everywhere. Then T must be periodic, so that there must be a periodic change of temperature, and it is easy to see that this may have great influence upon all solar phenomena. If such a period of eleven years exists, this would seem to account for the observed periodicity of solar activity. But this is not the only cause acting in this manner. In assuming ω to vary, we still retain the assumption that every point describes a circle, although with varying velocity. But some observations seem to speak for the existence of regular currents toward and from the solar equator, and probably there are still other motions combining with these. We only treat of the periodic changes in ω, because of the mathematical difficulties of the more general case, and because this suffices to indicate the essential principles involved.

It will, perhaps, be remarked that any such periodic changes ought to have been noticed if they are great enough to have such important consequences. But while it is almost certain from Auwers' investigations that the diameter of the Sun is not subject to great variations, the same cannot be said of the rotation law. This is an important subject for future investigation. An estimate which we will now begin to make of the magnitude of these variations, which would be necessary to explain the Sun-spot period, allows us to hope that we may, perhaps, be able to find these variations with our instrumental means.

Let T' and ω' be the values of temperature and velocity of rotation for the time t', if T and ω represent these quantities at the epoch t. Then assuming ρ and V to be unaltered

$$\omega'^2 - \omega^2 = \frac{1}{r}\left[c\,\frac{\delta\,(T' - T)}{\delta r} + \frac{c}{\rho}\,\frac{\delta \rho}{\delta r}\,(T' - T)\right].$$

We shall now, in our ignorance of solar temperature conditions, be forced to assume that when the temperature changes from T to T' it changes by the same number of degrees everywhere, so that

$$\frac{\delta\,(T'-T)}{\delta\,r} = \frac{\delta\,(T'-T)}{\delta\,z} = 0\ .$$

Then

$$\omega'^2 - \omega^2 = \frac{c}{r}\,\frac{1}{\rho}\,\frac{\delta\,\rho}{\delta\,r}\,(T'-T)\ .$$

$\frac{\delta\,\rho}{\delta\,r}$ is certainly very small near the surface. In order to be able to compute this expression, I will assume, as I have done in my thesis, that the density is inversely proportional to the square of the distance R from the center, so that

$$\rho = \frac{\sigma}{R^2}\ ,\quad R^2 = r^2 + z^2\ ,$$

where σ is a constant. Then

$$\omega'^2 - \omega^2 = -\frac{2\,c}{R^2}\,(T'-T)\ .$$

If $T' > T$, this gives $\omega'^2 < \omega^2$, *i. e.*, under these conditions a slower rotation corresponds to a higher temperature.

Suppose the gas to be hydrogen at the point where we are investigating its motion. For 0°C. and atmospheric pressure, *i. e.*, in absolute units, for $T = 273$, $p = 1013650$ dynes per square centimeter we have the density of hydrogen

$$\rho = 0.00008998 = 8998 \cdot 10^{-8}\ \left[\frac{gr}{cm^3}\right]\ .$$

Since $p = c\ \rho\ T$ we find

$$c = \frac{1013650}{273 \cdot 8988}\ 10^8\ .$$

The Sun's radius in centimeters is $R = 7.10^{10}$, so that

$$\omega'^2 - \omega^2 = -\ 17.10^{-13}\ (T'-T)$$

Let us suppose $T^1 - T = 100$, a change of temperature which may well be supposed to cause a considerable change in solar phenomena. Then

$$\omega^2 - \omega^{2\prime} = 17.10^{-13}$$

or

$$\omega - \omega' = \frac{17.10^{-13}}{\omega + \omega'} = \frac{17.10^{-13}}{2\,\omega}$$

if $(\omega - \omega^1)^2$ is neglected. Taking the equatorial rate of 862′ a day,

$$\omega = \frac{862}{86400} = 1'.10^{-2} = 3.10^{-6} \text{ radians per second.}$$

since $1' = 3.10^{-4}$ in a circle of radius unity. Therefore in one second

$$\omega - \omega' = \frac{17.10^{-13}}{6.10^{-6}} = 3.10^{-7}$$

and in one day

$$\omega - \omega' = \frac{864 \cdot 3.10^{-5}}{3.10^{-4}} = 86.'4$$

which is a very great difference between ω and ω^1. But if we remember that, in the regions where the spots are found, it is probably not hydrogen that plays the principal part, but the much denser metallic vapors, that the value we have taken for $\frac{\delta \rho}{\delta R}$ is almost certainly much too great, we shall find that the value of $\omega - \omega'$ just found is about a hundred times greater than that which we would have to expect upon the Sun. In fact, if the vapor is iron, we must divide the above number at least by 56. We can thus hardly expect differences of more than $1'$ in the daily arcs described by any point at different times. Moreover, we have only taken into account the variations in ω, while other disturbances of the regular circular motion may be much more important.

At all events it will be seen that the Sun-spot period does not appear quite as mysterious as it has done. We can even see how this may influence terrestrial magnetism. For, if the Sun, as many think, is a magnet or electromagnet, changes in the distribution of the Sun's temperature and density cannot but affect the magnetic or electromagnetic field surrounding it.

Much work is needed to decide whether the theory here offered is sufficient. The exact motion of the solar phenomena in latitude as well as in longitude must be investigated, and all the details of the rotation law must be closely studied. The rotation law is the key to a scientific theory of the Sun.

CHICAGO, August 30, 1897.

RADIATION IN A MAGNETIC FIELD.

By A. A. MICHELSON.

FURTHER analysis of the radiations emitted in a magnetic field shows that the phenomenon is much more complex than was supposed. An examination of the separate components of the "triplet" brings out the fact that in general these are multiple lines. The laws may be summarized as follows:

A.

1. All spectral lines are tripled when the radiations emanate in a magnetic field.

2. The separation is proportional to the strength of field and is approximately the same for all colors and for all substances.

3. Viewed in a plane perpendicular to the magnetic field the outer lines are polarized parallel with the field, and the central line perpendicular to the field.

4. Viewed in the direction of the lines of force, the central line vanishes, while the outer ones are circularly polarized, the component of shorter wave-length in the direction of the magnetizing current, the other in the opposite sense.

To these laws (which were verified by the examination of a dozen or more lines) the following must now be added.

B.

1. The "middle line" is a symmetrical triple, the distance between the components being one-fourth that of the "outer lines," and hence, also proportional to the strength of field.

2. The relative intensity of the components varies for different substances, and for different lines of the same substance; and accordingly the group may appear as a single line, or a double, or a triple.

3. The "outer lines" are unsymmetrical, but are symmetrically placed with respect to the "central line." The distance

between the components is usually one-fourth that between the "outer lines," but is in some cases one-sixth.

4. The intensity of the components varies for different spectral lines and these variations do not always correspond to those of the "central line." The outer groups may accordingly appear as single or double or triple or multiple lines.

FIG. 1 represents a plan of the arrangement of apparatus employed in the investigation. *S* is the source of light, either

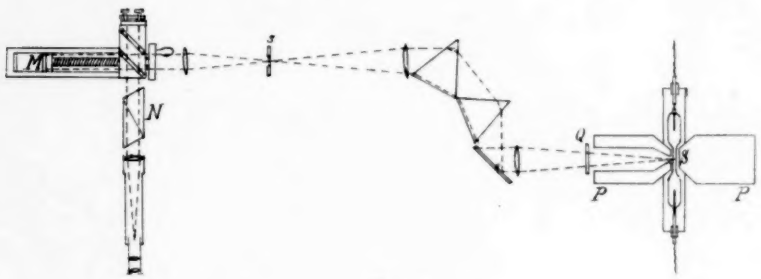


FIG. 1.

a small hand-blowpipe with a bead of the substance to be examined in the flame, or a vacuum tube which is usually placed in a metal box, for heating, of such form as to permit a close approach of the pole-pieces *P*, of an electromagnet. One of these is bored out to permit examination of the axial ray. The light from *S* undergoes a preliminary analysis by the spectroscopic train (two bisulphide prisms) the radiation to be investigated being isolated by the slit *s*. It then enters the interferometer, one of the mirrors of which, *M*, is moved on ways so accurately ground that no readjustment is necessary in any part of its path, that is, the mirror remains so nearly parallel with itself that the interference fringes (concentric circles) are always as clear as possible. The emergent beam then passes through the analyzer *N* to the observing telescope.

The clearness or "visibility" of the interference fringes is estimated at positions of the mirror *M* corresponding to increments of the difference of path of one, two, or five millimeters,

according to the nature of the curve. This, it must be admitted, leaves much to be desired in the way of precision, and in some cases there may be corrections of as much as 20 per cent. to reduce the observations to the value they should have, namely, $V = I_1 - I_2 / I_1 + I_2$, where I_1 is the maximum intensity and I_2 the minimum, for adjacent fringes. Doubtless much more accurate readings could be obtained by the use of comparison fringes,[1] but the process is so much more tedious that the form of the curve is liable to alter during the observations. The case is somewhat analogous to eye estimations of stellar magnitudes, which are but little inferior to photometric determinations and much less troublesome.

In any case it is always easy to distinguish ascending and descending slopes, and maxima and minima can be located with very great accuracy, and this is usually quite sufficient to permit a fairly accurate deduction of the distribution of light in the spectrum. It has been shown[2] that with the definition of visibility just given, if $y = \phi(x)$ is the intensity curve of the spectrum

$$P\,V = \sqrt{C^2 + S^2} \text{ in which}$$

$$P = \int \phi(x)\,dx,\ C = \int \phi(x) \cos kx\,dx, \text{ and } S = \int \phi(x) \sin kx\,dx,$$

the integration extending over the spectrum.

But by Fourier's formula

$$\phi(x) = \int_0^\infty C \cos kx\,dk + \int_0^\infty S \sin kx\,dk$$

so that if C and S are both known $\phi(x)$ can be determined. In general this is not the case unless another relation between C and S is given. Such a relation is furnished by the "phase curve" which gives the displacement of the fringes from the position they would occupy had the source been homogeneous. If δ is this displacement and $\theta = 2\pi\delta/\lambda$ then

$$C = V \cos\theta \text{ and } S = V \sin\theta.$$

[1] See *Phil. Mag.*, September 1892. [2] *Ibid.*

In general the θ curve is troublesome to observe on account of the difficulty in securing a sufficiently homogeneous comparison source, but in the present instance this is furnished by the non-magnetized radiations.[1] Usually, however, the assumption was made that the spectrum was symmetrical, and in only a few cases was the solution verified by the complete analysis. In this simpler form we have

$$\phi(x) = \int_0^\infty V \cos kx \, dk.$$

This integral may frequently be calculated when V can be expressed in simple analytical form as a function of k. In general this is not the case, and it was for the solution of such problems that the harmonic analyzer[2] was devised. The curve, $V=f(k)$ is "fed" to the machine, which then draws the curve $y=\phi(x)$, the whole operation taking but a few minutes.

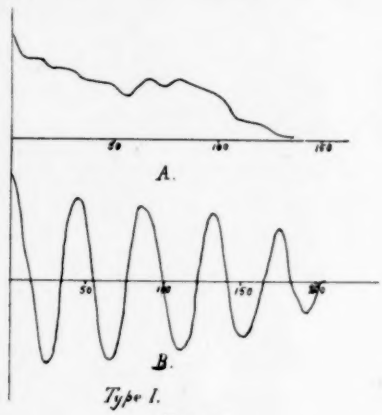


FIG. 2.

It was found on completing the analyses of some fifty or more visibility curves, that the resulting spectra could be classified under three types; there were some interesting variations which would merit a separate investigation, but most of the cases could be identified at a glance.

[1] These are not always sufficiently simple, as in the case of the green thallium line.

[2] *Phil. Mag.*, January 1898.

The three types of visibility curve are given in Figs. 2, 3, and 4. Those marked *A* referring to observations made with the

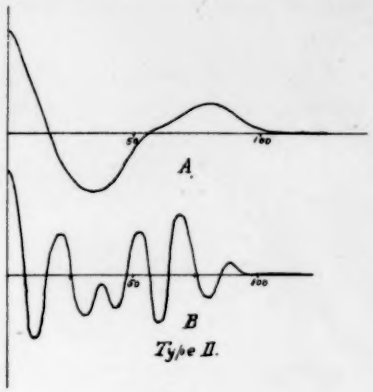


FIG. 3.

line of sight at right angles with the magnetic field and with the plane of polarization perpendicular to the lines of force; while *B* correspond to observations with the line of sight still

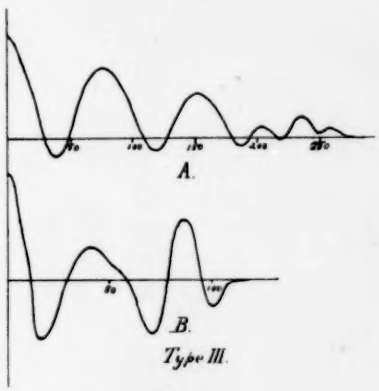


FIG. 4.

normal to the field, but plane of polarization parallel with the lines of force.

It was found that there was no appreciable difference between these last, and the observations with the line of sight parallel

with the field; but in this case it was possible to analyze either one of the outer groups separately, by the use of the quarter-wave plate Q, Fig. 1. This was done in a few cases, but no new result was obtained.

The abscissæ of the visibility curves are differences of path in millimeters, reduced to a field strength 10,000, as determined by a bismuth spiral.

Fig. 5 gives the intensity curves of the corresponding spectra, the abscissæ being in tenth-meters.[1]

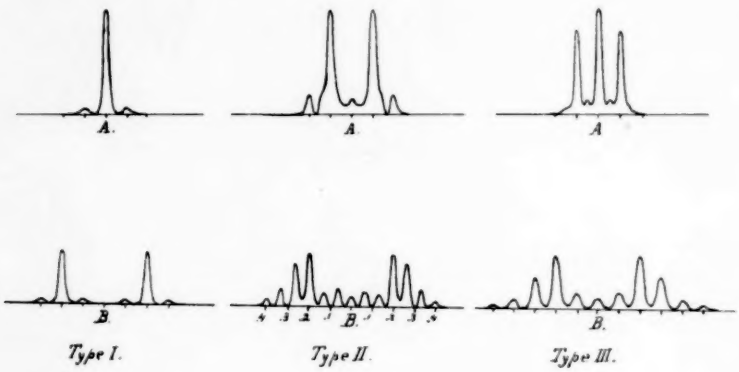


FIG. 5.

The following is a list of the radiations and their classification.

Mercury:	yellow lines	type I
	green line	type III
	violet line	type II
Cadmium:	red line	type I
	green line	type III
	blue line	type II
Zinc:	red line	type I
	green line	type III
	blue line	type II

[1] For this the abscissæ of the curve drawn by the analyzer are multiplied by the square of the wave-length.

Sodium:	yellow lines	type II
Thallium:	green line	type II (doubtful)
Lithium:	red line	broadened
Hydrogen:	red line	broadened
	blue line	broadened
Helium:	yellow line	broadened
	green line	broadened

The following table shows that the law *A* 2 is only approximately true. In fact, owing to the complexity of the spectra there is considerable latitude in the choice of the distance between the outer groups. If this correspond to the brightest components the law can hardly be said to hold at all; but if the distance be taken between the centers of gravity of the light areas, a fair agreement is found. The table gives separation in tenth-meters for a field 10,000. The lines marked with an asterisk are less accurate than the others on account of broadening.

* Hydrogen	red	0.48
* Lithium	red	0.60
Cadmium	red	0.42
Zinc	red	0.42
Mercury	yellow	0.36
* Sodium	yellow	0.50
* Helium	green	0.37
Mercury	green	0.40
Cadmium	green	0.41
Zinc	green	0.40
* Thallium	green	0.36
Cadmium	blue	0.40
Zinc	blue	0.33
Mercury	violet	0.33

Taking into account the uncertainty alluded to, the results show on the whole a fair agreement, from which it may be concluded that the separation is independent of the radiating substance and of the color.

It is possible that some of the resemblances in the preceding tables are due to the fact that the substances in question are chemically related, and perhaps it is scarcely justifiable to generalize from such a limited number; and it may well be that a wider range of elements would show other peculiarities.

I desire to express my hearty appreciation of the efficient service rendered in this work by Mr. C. R. Mann, and especially to recognize the patience and skill shown in the tedious and delicate process of preparation of the vacuum tubes, to which in great measure the success of the investigation is due.

RYERSON PHYSICAL LABORATORY,
January 1898.

Minor Contributions and Notes.

A VARIABLE BRIGHT HYDROGEN LINE.[1]

The presence of the bright hydrogen line $H\beta$ in the spectrum of the star *A. G. C.* 9181 was found from the Draper Memorial photographs in 1895 and was announced in the Astrophysical Journal, 1, 411. From a comparison of photographs of this object taken on different dates Miss A. J. Cannon finds that this line is variable. On October 5, 1892, it was invisible. On November 28, 1894, it was about half as bright as the corresponding line in *A. G. C.* 9198, ω Canis Majoris. On April 27 and 30, 1895, the line in *A. G. C.* 9181 was distinctly the brighter of the two, while in January 1897, it was again invisible. From a large number of photographs of this object taken recently it appears that this line, which was bright in October 1897, is now, December 27, invisible.

Edward C. Pickering.

January 1, 1898.

A NEW SPECTROSCOPIC BINARY.[1]

From an examination of the Draper Memorial photographs Mrs. Fleming finds that the star *A. G. C.* 20263, β Lupi, is a spectroscopic binary. The period has not yet been determined but photographs are being taken for this purpose.

Measures of the spectroscopic binaries μ^1 Scorpii and *A. G. C.* 10534 show that the relative velocities of the components are approximately 460^{km} and 610^{km} respectively. The velocities are, therefore, much greater than in the case of ζ Ursae Majoris and β Aurigae. The separation of some of the lines amounts to as much as nine tenth-meters.

Edward C. Pickering.

January 1, 1898.

PHOTOGRAPHIC MAGNITUDES.[2]

In determining the photographic magnitudes of the stars it is a matter of great importance to know how much their relative brightness will

[1] *Harvard College Observatory Circular No. 21.* [2] *Ibid., No. 22.*

vary on different plates, or on different portions of the same plate. It is especially important to determine the amount of this error, since it is not easily eliminated and has been supposed to be large by some persons not familiar with stellar photographs. A moment's investigation of photographs of the same portion of the sky shows that this source of error is small, so small that it is not readily determined by direct measurement. The uniformity of different portions of the film is shown by allowing the stars to trail over the plate. The different portions of the trails appear equally intense, and no variation is perceptible to the eye. A much more delicate test was found in the discussion of a series of measures, made by Miss E. F. Leland, of the variables discovered by Professor Bailey in the cluster Messier 5. Sixty-three of these variables were compared on 41 plates by Argelander's method, with a sequence of comparison stars. Estimates were made of the difference in grades of each variable from the next brighter and the next fainter star of the sequence. The sum of these differences gives the interval between the comparison stars and combining all the results gives, in general, several measures of each interval on each plate. Each comparison star in turn may then be regarded as a variable and its changes in light determined from the next brighter and next fainter star of the sequence. Comparatively few measures were made of the six brightest and the three faintest stars of the sequence. The five intermediate stars were measured on 41, 39, 38, 30 and 30 photographs respectively. The corresponding ranges in the measures derived from each plate, none being rejected, were 0.14, 0.10, 0.12, 0.15, and 0.08 magnitudes, and the average deviations, 0.02, 0.01, 0.02, 0 03 and 0.02. The largest residual was 0.10 and this depended upon two estimates only. We find, therefore, that on the average, five stars were measured on 35 plates with a range of 0.12 magnitudes and an average deviation of 0.02 magnitudes. The total number of estimates from which these results are derived is 4294. The average deviation 0.02 includes: (1) The errors of observation, which are increased by the fact that four estimates enter into each determination, but are diminished since on the average twelve determinations were made of each interval on each plate. (2) Errors due to neglecting hundredths of a magnitude, the computation so far being made only to tenths. (3) Errors due to irregularities in the film, which enter with their full value into the result. Since the combined effect of these three sources of error is only $\pm$ 0.02, it is evident that neither of them can be large. The errors due to the film are in fact so small that there is no

evidence that they exist and more delicate methods of measurement are required to render them perceptible.

EDWARD C. PICKERING.

January 4, 1898.

THE VARIABLE STAR U PEGASI.[1]

MUCH difficulty has been experienced in determining the nature of the variations in the light of this star. Mr. Chandler states that he at first supposed that it was a star of the Algol type having a period of $2^d.06$ or $2^d.07$. It was then observed by Mr. Yendell, who confirmed the variability and still regarded it as of the Algol type, but with a period of $0^d.69$. Mr. Chandler, under date of October 26, 1895, announces in the *A. J.*, **15,** 181, that the period is $5^h\ 31^m\ 9^s.0$ which "is probably only a moderate fraction of a second in error," that it is not of the Algol type, but that the times of increase and decrease are equal. Mr. Yendell (*A. J.*, **16,** 78) states that the time of increase varies from $1^h\ 28^m$ to $3^h\ 41^m$. Again, Mr. Chandler (*A. J.*, **16,** 107) announces that the period is $5^h\ 32^m.25$, that it is perfectly regular, and that previous discrepancies are due to a large subjective error amounting to $0^m.6p$, in which p is the parallactic angle. The correction for this error will sometimes increase and sometimes diminish the observed time of minimum by as much as half an hour. He also states that the decrease in light is more rapid than the increase, and, referring to the short period variables η Aquilæ and δ Cephei, that "we may, therefore, regard U Pegasi, provisionally, as a type of variability distinct from this class of stars, as it evidently is from those of the Algol type."

Owing to these uncertainties it seemed desirable to determine the true form of the light curve photometrically, especially as such observations are free from the subjective error mentioned above, since the images compared are constantly interchanged. Assuming the light curve to be constant, it appeared possible to determine its form from observations made during a single evening. Accordingly, measures were made by Mr. O. C. Wendell with the polarizing photometer (this JOURNAL **2,** 89) attached to the 15-inch equatorial telescope of the Harvard College Observatory. The results for a single evening, December 28, 1897, are contained in the annexed table, the first column giving the Greenwich Mean Time, and the second the pho-

[1] *Harvard College Observatory Circular No. 23.*

tometric magnitude found by adding the observed difference in magnitude between U Pegasi and the comparison star, + 15°4916, to 8.90, the photometric magnitude of the latter star. These observations are also indicated by the crosses in the figure. The third column gives the residuals found by subtracting from the magnitude given in the second column that derived graphically from all the observations.

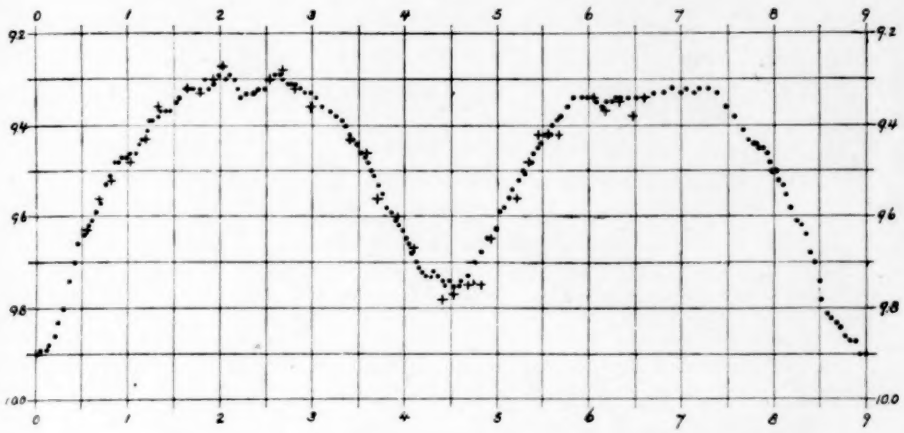
OBSERVATIONS ON DECEMBER 28, 1897.

G. M. T.	Mag.	O—C	G. M. T.	Mag.	O—C
10h 26m.9	9.63	.00	13h 48m.7	9.60	—.01
10 34 .2	9.56	—.01	13 58 .6	9.67	—.02
10 42 .2	9.52	+.01	14 17 .5	9.78	+.04
10 55 .3	9.48	+.02	14 24 .5	9.77	+.02
11 4 .7	9.43	+.02	14 34 .0	9.75	+.02
11 12 .4	9.36	—.02	14 43 .0	9.75	+.07
11 31 .3	9.32	—.01	14 50 .0	9.65	.00
11 40 .0	9.33	+.02	15 7 .0	9.56	+.03
11 48 .6	9.30	.00	15 15 .0	9.48	.00
11 55 .4	9.27	—.02	15 21 .0	9.42	—.02
12 25 .8	9.30	.00	15 26 .8	9.42	.00
12 33 .4	9.28	—.02	15 33 .2	9.42	+.04
12 41 .2	9.32	+.01	15 56 .9	9.34	.00
12 51 .1	9.36	+.03	16 3 .0	9.37	+.02
13 17 .4	9.43	+.01	16 13 .3	9.35	+.01
13 27 .2	9.46	—.01	16 21 .2	9.38	+.04
13 36 .0	9.56	+.03	16 29 .6	9.34	.00

Each observed magnitude is derived from the mean of four sets of four settings each, the photometer being reversed between the second and third sets, and the positions of the images of the stars interchanged after the second setting of each set. The magnitudes derived from each pair of sets differ on the average ± 0.04 mag. The average values of this quantity during the seven successive hours of observation were 0.06, 0.03, 0.04, 0.03, 0.04, 0.04, and 0.04, respectively. As they show no progressive increase it is evident that the fatigue of the observer did not sensibly affect the accuracy of the observations. The accuracy is also unaffected by the altitude, which was only 5°.6 at the end of the series. No correction has yet been applied for differential absorption, which is small but sensible, although the distance between the stars compared is only 15′. The computed probable error of a single set of four settings from these differences is ± 0.024 mag.

The average value of the residuals in the third column is ± 0.017, only one being greater than 0.04.

It is obvious from an inspection of these observations that the star had nearly the same brightness after an interval of $4^h\ 30^m$, at first



indicating that the variable had this period and not $5^h\ 31^m$. But later observations showed that the phenomena were not quite so simple. Assuming the period $4^h\ 30^m$ it soon appeared that the even minima were fainter than the odd. Observations continued through even minima on four nights gave the minimum brightness of U Pegasi, including errors of observation, 9.94, 9.94, 9.92, and 9.85, respectively, while corresponding measures of odd minima gave the magnitudes 9.78, 9.78, and 9.75. It thus became evident that the period should be doubled, and plotting the observations a light curve was formed closely resembling that of β Lyrae. The formula *J. D.* 2413514.6157 + $0^d.37478E$ has been adopted, which closely represents the observations recently made and also a few less accordant observations made here in 1895. Changing the period by two seconds, to $0^d.3748$, would alter the times of minima in 1895 by about an hour, while the uncertainty is only a small fraction of this amount. The period may, therefore, be assumed to be $8^h\ 59^m\ 41^s$. The magnitude at maximum is 9.30, at primary minimum 9.90, and at secondary minimum, 9.75.

The above figure represents the observations on a scale such that in the ordinates one division corresponds to a tenth of a magnitude,

and in the abscissæ to 30 minutes. All the observations made by Mr. Wendell on eight nights are included, none being rejected for discordance. In fact, the most discordant set of sixteen settings differs from the mean curve by less than a tenth of a magnitude. Each dot represents 80 settings, the method of overlapping means, in common use in meteorology, being employed. The observations of December 28, as stated above, are represented by crosses, each representing the result of sixteen settings. The total number of settings was 2784 and the entire time of observation, including rests, about 30 hours.

If we neglect the difference between the primary and secondary minima, reduce the half period to fractions of a day, and multiply it by 16 we obtain the product $2^d.99824$, or very nearly three days. Therefore, the phases recur at nearly the same times every three days. If we multiply the period $5^h\ 32^m\ 15^s$ by 13 we obtain $2^d.99948$, or very nearly the same quantity. In either case, therefore, the phases would recur at the same times every three days, but with the second period the interval between successive minima would be greater than with the first period in the ratio of 16 to 13. If then we used the second period we should find the observed minima in error by an amount equal to three-thirteenths of the hour angle, or expressed in minutes $0^m.92h$, in which h is the hour angle expressed in degrees. Since the latitude of Cambridge is $+\ 42°.4$ and the declination of U Pegasi is $+\ 15°.4$ it follows that when h is small we may write $h = 0.61p$ and $0^m.92h = 0^m.56p$, which agrees very nearly with $0^m.6p$, the correction found empirically by Mr. Chandler from his observations. The coefficient becomes larger with large hour angles having the values 0.61, 0.74, and 0.91 for $h = 15°$, $30°$, and $45°$, respectively. It therefore follows that by the application of the correction $0^m.6p$ not only were the observed times of minima changed in some cases by more than half an hour, but an error of more than an hour was introduced into the half period. An arbitrary correction proportional to the parallactic angle is not to be recommended, since by assuming different values of the coefficient, very different values of the period will be found.

From the above discussion it appears that U Pegasi is no longer the variable star having the shortest period known. This position appears to be held by the variable ω Centauri 19, discovered by Professor Bailey, who finds its period to be $7^h\ 11^m$. Although U Pegasi can no longer be regarded as an example of that peculiar class of short

period variables having a single maximum in which the decrease is more rapid than the increase, this class is still represented not only by S Antliæ, but by ω Centauri 24, which Professor Bailey finds to decrease twice as rapidly as it increases, while ω Centauri 45 increases at least five times as fast as it diminishes.

EDWARD C. PICKERING.

January 14, 1898.

REPORT ON A NEW EDITION OF THE NORTHERN DURCHMUSTERUNG CHARTS.[1]

(From a communication by Dr. F. Küstner.)

I.

WITH regard to the *first* edition, the following information has been obtained:

1. The original stones have not been preserved. Probably only a few stones were used and repolished for different charts.

In the case of the SD. the stones are preserved.

2. The contract between Argelander and the publisher, Marcus, made the latter the proprietor of the charts, although the government paid one-half of the expenses of engraving and printing.

3. The *Northern Durchmusterung* was sold for ninety marks, but the publication was not lucrative. In the case of the SD. there has probably been a loss, because, within ten years of the publication of the charts, less than 100 copies were sold, and at present the sales average about two copies a year.

II.

About the *new* edition, the following facts are reported:

1. While the Observatory at Bonn has no rights in the new edition, it considers it a point of honor to coöperate with the publisher in making the edition free from error as far as possible. For this purpose an extended list of errors has been compiled, and the corresponding corrections have been made on the charts wherever they would change the configuration of the stars. On some charts the number of corrections amounted to about fifty. Notice of erroneous or doubtful stars will be gladly received, and the matter will be looked up in the original records, as has been the custom heretofore at the Bonn Observatory.

[1] Report of a committee appointed at the Yerkes Observatory Conferences.

2. From the many experiments made at Bonn, it was found that while the mere reproduction of the charts was easy, it was very difficult, technically, to get prints which were free from specks or spurious stars.

3. The actual printing of the charts had been going on for about a year when the printing firm failed. Dr. Küstner says that a year's labor in reading and correcting the proofs has thus been lost.

4. Flittner, Marcus' successor, is trying to engage another printing firm, and is now corresponding with the government printing office at Berlin. It is the publisher's intention to call for subscriptions as soon as a sufficient number of some of the charts has been reproduced. The selling price will depend on the number of subscribers, and, according to Dr. Küstner's view, the coöperation of our committee should be directed principally towards making this number as large as possible.[1]

J. G. HAGEN, S. J.

GEORGETOWN COLLEGE OBSERVATORY,
January 6, 1898.

A NOTE ON THE FIGURING AND USE OF THE ECCENTRIC AND UNSYMMETRICAL FORMS OF PARABOLIC MIRRORS.

IN a valuable paper which is published elsewhere in the present number of this JOURNAL[2] Professor Poor has investigated in a very complete manner the question of the spherical aberration at various points in the field of a parabolic mirror, and has derived general equations which determine rigorously the diameter (maximum) of a star image in the principal plane of the parabolic section. As I have pointed out elsewhere, these results, although valuable in themselves, do not, when taken alone, enable us to draw any very complete or definite conclusions as to the definition and resolving power of mirrors

[1] Since the above report was put in type circulars asking for subscriptions to the new edition of the charts have been received from the publishers. It is stated that the new edition will be published provided at least one hundred copies of the Atlas are ordered by May 1, 1898. As the price is only 70 marks, and as every astronomer must feel it a duty to assist in bringing these invaluable charts within the reach of all observers, there should be no difficulty in securing the requisite number of subscriptions. Intending subscribers should address A. MARCUS UND E. WEBER'S VERLAG, BONN.—EDS.

[2] See p. 4.

in different portions of the field, because these qualities are determined more by the *effective distribution of light* in the image than by the mere linear extent or "spreading" of the latter in the focal plane.[1] In a general way, however, they support the conclusions reached by the writer on quite different grounds of comparison[2] that mirrors are inferior to the ordinary reflector, and the photographic doublet in the respective fields of visual observations and general photographic work. But it is not in either of these fields that mirrors should be employed if their peculiar excellencies are to be best utilized and their peculiar defects minimized in importance. Reflecting telescopes are, as has often been pointed out,[3] preëminently adapted to stellar spectroscopic and allied branches of astrophysical work, and it is to such work (which is now certainly important enough in itself to warrant the construction and erection of large telescopes especially designed for its most efficient prosecution) that they should therefore be devoted.

Now in stellar spectroscopic work we have to deal only with the image of a point at or very near the center of the field, where the definition is sensibly perfect with any form of mirror, and the particular objections, on theoretical grounds, against even the highly eccentric form (90°) considered by Professor Poor, vanish. Could the practical difficulties of figuring the surface be overcome, this latter form would, on account of the simplicity and beauty of the mounting, particularly the small number of reflections involved in making the instrument of the *coudè* form, be of the greatest value in astrophysical work. These difficulties, however, are most formidable, although I am inclined to think after discussing the matter with Professor Poor, that they may be overcome by special methods. It is to be greatly

[1] See *Pop. Astron.* **5**, 528, Feb. 1898. In this paper the general equations were given which enable us to determine the distribution of light in the image of a star or other point source at any part of the field of a parabolic reflector.

[2] See papers in *Pop. Astron.* **5**, 200, August 1897; *A. N.*, No. 3439; *Knowledge*, August and September 1897, pp. 193, 218; *Obs'y*, **20**, 303, 365, 404, September, October, and November 1897.

[3] See paper by HALE "On the Comparative Value of Refracting and Reflecting Telescopes for Astrophysical Investigation," *Ap. J.*, February 1897, p. 119; also various papers by the writer—in the *Phil. Mag.*, July and October 1894, pp. 137 and 337; *Astron. and Astrophysics*, December 1894, p. 835; *Ap. J.*, January 1895, p. 52, March 1894, p. 232, November 1894, p. 264, March 1896, pp. 169, 182, 183, May 1896, p. 347, November 1896, p. 274, February 1897, p. 132, etc.

hoped, therefore, that Professor Poor will continue his experiments in this direction and succeed in carrying them to a successful issue.

In this connection it might not perhaps be out of place to refer to certain suggestions of my own of somewhat the same character as that of Professor Poor. Some time ago I proposed to use mirrors cut from one side of a paraboloid of revolution for use in certain forms of astronomical and laboratory spectroscopes and spectrobolometers and in the spectroheliograph.[1] In these cases the mirrors required are small, and no difficulty would be experienced in obtaining them, figured with as great accuracy as desired, by cutting them out of one side of a larger paraboloid of revolution figured and polished by the usual method. The portion cut out and used would moreover be quite near the center of figure (only far enough to one side to allow the incident beam to clear the slit-plate, observing eyepiece or bolometer strip), and the size of the original mirror would not therefore need to be much more than twice the diameter of the part utilized. Such a figured surface would furnish at least four smaller mirrors of the form required. I have also considered the possibility of grinding a single large disk for a telescope objective so that the center of the parabolic figure is at, or near, one edge, instead of at the center of the disk. This might perhaps be accomplished by present methods, by embedding the glass disk eccentrically in a suitable matrix of the same general nature and hardness as the glass itself (perhaps furnace slag or broken glass, finely ground and then recemented together, under pressure in a mold of proper shape, would be found to answer well), and then figuring the whole surface into a paraboloid of revolution as usual.

Such a speculum, if it could be made, would realize the advantages of increased light-gathering power possessed by the Herschelian form of reflecting telescope without the disadvantage of working considerably out of the optical axis, as is necessary in the latter form of instrument.

It will be readily seen, from the preceding considerations, that, in the opinion of the writer, the question of the aberration of mirrors for points some distance from the optical axis is of comparatively little practical importance; only the center of the field being used in those lines of work for which reflecting telescopes are best adapted. The investigation of this question is, nevertheless, of great theoretical interest, and I am very glad to see that Professor Poor has taken it up.

[1] See papers in ASTROPHYSICAL JOURNAL already referred to.

The results he has already obtained (for the size of the distorted image), when supplemented by the investigation which he proposes to undertake[1] of the distribution of light in the image will furnish us for the first time with data which will make it possible to draw definite conclusions as to the practical defining and resolving powers of various forms of parabolic mirrors at points in the field some distance from the optical axis.

F. L. O. WADSWORTH.

YERKES OBSERVATORY,
Dec. 8, 1897.

[1] See footnote on page 119.

Reviews.

Ueber einige Emissionsspectra des Cadmiums, Zinks, und der Haloid Verbindungen des Quecksilbers und einiger anderen Metallen. A. C. Jones. *Wied. Ann.* **62**, 30–53, 1897.

The Multiple Spectra of Gases. John Trowbridge and Theodore Wm. Richards. *Am. Jour. of Sci.*, **3**, 117–120, 1897.

On the Propagation of Waves along Connected Systems of Similar Bodies. Lord Rayleigh. *Phil. Mag.*, **44**, 356–362, 1897.

On a Relation between the Spectrum of Hydrogen and Acoustics. A. S. Herschel. *The Observatory*, June 1896.

Of the known spectroscopic properties of the elements, two of extraordinary importance and of increasing interest have been established within the last third of the century. One of these is the fact that, under different physical conditions, the same element yields radically different spectra; the other is that, under the *same* physical conditions, many, possibly each, of the elements yield "series" of vibrations, *i. e.*, vibrations which are related to one another in a definite, and apparently simple, manner.

Neither of these phenomena have yet been "explained;" by which we mean, in the first case, that the exact physical conditions under which different spectra of one and the same element are produced, have not yet been "described;" and in the second case, that the mechanical analogue of a system vibrating in the "series" mode has not yet been "described."

Professors Trowbridge and Richards have made a long step in showing that the oscillatory discharge gives one class of spectra, the deadbeat discharge another; but, even yet, we are left in the dark as to whether this change of spectrum is due more immediately to a change of temperature, or to some chemical change which is wrought by the one discharge and not wrought by the other; or as to whether it may not be due to still other causes, perhaps mere changes in the time and space distribution of electric potential in the luminous gas. And still another alternative must be mentioned. For so many groups and sub-

groups of lines have been distinguished in the spectra of single elements, that it is not impossible that we may have purely thermal, purely electrical, and purely chemical spectra overlapping, and *present together* in the ordinary spark or arc-spectrum.

In illustration of these groups and sub-groups, I need only mention lines which fall into series and lines which do not (Kayser and Runge); hydrogen of Balmer's series and hydrogen in ζ Puppis (Pickering); lines which "shift" under pressure and lines which do not (Humphreys and Mohler); lines which depend upon the presence of another element and lines which do not, *e. g.*, aluminum and oxygen; lines which are strong in the arc-spectrum and weak in the spark-spectrum; lines which are rapid in making their appearance in the Plücker tube and lines which are less rapid (Schuster, *B. A. Report*, 1897).

In the first of the articles under review, the author, Mr. A. C. Jones, has undertaken to discover just what effect is produced upon the spark-spectra of zinc, cadmium, mercury and their respective halogen compounds when the electric discharge varies in *intensity* but not in *kind*.

This variation of intensity is accomplished either by narrowing the bore of the vacuum tube, or by introducing a spark gap, or by the use of both methods.

The changes in intensity of the lines corresponding to changes in intensity of the spark-discharge are summarized in tables.

This interesting paper is marked by one error whose very frequency of occurrence calls for comment in this place, *i. e.*, the identification of *resolving power* of the spectroscope (auflösende Kraft, p. 35) with the *scale* of the spectrum.

Lord Rayleigh has pointed out in the most unmistakable manner that the resolving power of a prism cannot be described without giving the difference of thickness of prism traversed by the two extreme rays, or giving some statement which is mathematically equivalent.

The beautiful investigation of Professors Trowbridge and Richards regarding the red and blue spectrum of argon has been extended so as to include nitrogen, hydrogen, and helium. The following single sentence states the purpose of the work and summarizes their results. "It is the object of this paper to emphasize anew the importance of the electrical conditions of the circuit, and to call attention once more to the fact that the behavior of most elementary gases is in every respect similar to that of argon." That is to say, in general, the oscil-

latory discharge produces the blue and linear spectra, while the dead-beat discharge gives the red and channeled spectra.

Some exceptions to this statement are most instructive, *e. g.*, "no considerable effect" is produced upon the spectrum of helium when the discharge is changed from the oscillatory to the continuous.

Again, if the continuous discharge be produced by placing impedance in a circuit which is already oscillatory, the spectrum obtained differs very markedly from that given by the battery alone without any condenser in the circuit.

As a further extension of this work, the authors propose to use their powerful battery to determine "whether the oscillatory discharge produces its effect simply by increasing the temperature, or because of some inherent property in the manner of discharge." An experimental answer to this very fundamental question will be awaited with unusual interest; for such an answer is one of the great needs of modern spectroscopy.

The next paper, that of Lord Rayleigh on vibrating systems, has a most important bearing on the second of the general problems stated at the outset.

It is a study of the modes of vibration of a system of similar bodies distributed at equal intervals, either along a straight line of infinite length, or in a closed chain.

By the use of generalized coördinates, the equations of motion are elegantly derived in a linear form with constant coefficients. Following the general solution is a study of some half dozen special cases, in each of which the object is to express the frequency in terms of the wave-length; or, what is the same thing, since in all cases $v = n\lambda$, to determine the relation between speed and wave-length.

Among these special cases is a generalized expression of the ordinary formula for the frequency of a stretched string. This generalized expression includes also Lord Kelvin's wave model as a special case.

A linear system of magnets — a row of compass needles — is next considered. Following this is Fitzgerald's system of pulleys belted together by means of rubber bands. The last case considered is that of an open chain of magnets for which the frequency, n, is found to be

$$n = 2 \cos \frac{s\pi}{2m} = 2 \cos \frac{a\pi}{2\lambda}$$

where a is the distance between the successive magnets, λ is the wave-

length, and m the total number of magnets in the chain, s being some integer.

If N be the highest possible frequency of the system (corresponding to a configuration in which the magnets are all parallel to each other) the above expression takes the form

$$n = N \cos \frac{s\pi}{2m} = N\left(1 - \frac{s^2\pi^2}{8m^2}\right)$$

where, if s (the number of waves in the length of the chain) is made to vary, the frequency will vary in the same general way as in a spectral series.

But the most interesting feature in the whole discussion, from the spectroscopic standpoint, is that when s is held constant, say $s=2$, and m is varied, the frequencies are distributed in a series almost identical with that of Balmer, which is as follows:

$$n = N\left(1 - \frac{4}{m^2}\right).$$

But when m assumes different integral values, beginning with 3, we have for each different value of m a different mechanical system. This allotment of each line in the series to a different, if not an entirely independent, dynamical system is an entirely new and suggestive point of view.

One's hopes for immediate progress along this line are, however, somewhat dampened by the closing paragraph, which must be quoted in full.

"There is one circumstance which suggests doubts whether the analogue of radiating bodies is to be sought at all in the ordinary mechanical or acoustical systems vibrating about equilibrium. For the latter even when gyratory terms are admitted, give rise to equations involving the square of the frequency; and it is only in certain exceptional cases, *e. g.*, (31) that the frequency itself can be simply expressed. On the other hand, the formulæ and laws derived from observation of the spectrum appear to introduce more naturally the *first* power of the frequency. For example, this is the case with Balmer's formula. Again when the spectrum of a body shows several doublets, the intervals between the components correspond closely to a constant difference of frequency, and could not be simply expressed in terms of squares of frequency. Further the remarkable law discovered independently by Rydberg and Schuster connecting the con-

vergence frequencies of different series belonging to the same substance, points in the same direction.

"What particular conclusion follows from this consideration, even if force be allowed to it, may be difficult to say. The occurrence of the first power of the frequency seems suggestive rather of kinematic relations[1] than those of dynamics."

To those who are accustomed to look upon work of this kind as dreamy speculation we recommend a careful and minute reading of the original paper, in confidence that the reader will there find only a very accurate and very beautiful description of the behavior of some six or eight actual dynamical systems.

The avowed object of the next paper is to produce an acoustical, and hence mechanical, analogue of Balmer's series. While more speculative than the preceding, it is full of suggestion.

The author first points out that if, while an open organ pipe be sounding the m^{th} harmonic of its fundamental, a wind current be driven through the pipe, the resonance for its m^{th} harmonic will be destroyed: and if the speed of the wind be the $\left(\frac{n}{m}\right)^{\text{th}}$ part of the speed of sound in air, the vibration will no longer be the m^{th} harmonic, but will have a frequency related to that of the m^{th} harmonic as follows:

$$\frac{1}{\lambda} = A\left(1 - \frac{n^2}{m^2}\right).$$

For a wind whose speed is $\left(\frac{2}{m}\right)^{\text{th}}$ that of sound, we have

$$\frac{1}{\lambda} = A\left(1 - \frac{4}{m^2}\right).$$

But one is not to be deceived by the outward resemblance between this expression and that of Balmer; for the limiting frequency, A, is here *not* a constant, as in Balmer's expression, but it is itself a function of m, viz., the product of m by the frequency of the fundamental.

Accordingly the author takes refuge in a more complex atom. He imagines the open organ pipe replaced by a circular tube—a hollow annulus—in which the wind current is replaced by a current of ether, flowing with a speed which is $\left(\frac{2}{m}\right)^{\text{th}}$ that of light. These annuli are then

[1] *E. g.*, as in the phases of the Moon.

assembled to form a vortex-ring atom, each stream-line of the ordinary vortex ring being replaced by one of these resonant ring tubes.

If now we *assume m* to vary from one tube of flow to another, and if we *assume* farther that each annulus can respond to and emit *one harmonic only*, then we have a fair approximation to Balmer's series.

But there is here evidently much of speculation and we are no longer on the solid foundation of experiment and mathematical deduction. H. C.

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

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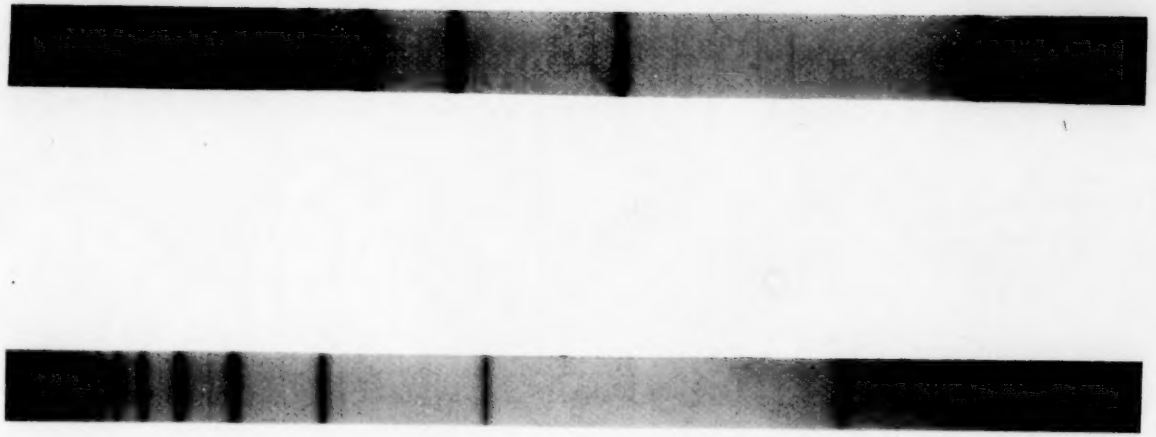
PLATE III.



PHOTOGRAPHS OF THE SPECTRUM OF SIRIUS TAKEN WITH A CONCAVE GRATING.